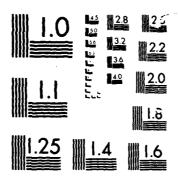
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FOUNDATION INTERACTION PROBLEMS INVOLVING AN ELASTIC HALF-PLANE

by

Howard B. Wilson, Louis H. Turcotte

University of Alabama University, Alabama 35486



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Stress analyses stemming from the interaction of a structure and its foundation are the basis for this report. Often times such problems create voluminous costs and efforts. Here, a more efficient, both money and labor wise, procedure is presented.					

Exact equation solutions for an elastic half-plane exposed to arbitrary

surface loading are computed for more satisfactory answers to this broad

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20. ABSTRACT (Continued).

Several formulations are shown to yield solutions that are, in turn, used to compute stiffness matrices and further implemented to investigate other types of interaction problems.

The Euler beam, resting on an elastic half-plane and subjected to external loading, is a simple type interaction problem. Such foundation interaction forces and other beam shear and moment quantities are likewise computed.

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PREFACE

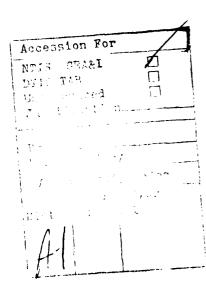
This report presents a computationally efficient procedure for solving soil-structure interaction problems involving an elastic half-plane. The procedure employs the exact solution of the equations of elasticity for an elastic half-plane. The work in preparing this report and computer program was accomplished with funds provided to the US Army Engineer Waterways Experiment Station (WES), Vicksburg, Mississippi, under the Civil Works Research and Development Program of the Office, Chief of Engineers (OCE), US Army, as part of the Structural Engineering Research Program work unit of the Soil-Structure Interaction (SSI) studies project.

This report and computer program were written by Dr. Howard B. Wilson and Mr. Louis H. Turcotte of the University of Alabama, under Contract No. DACW39-83-M-2630 with WES.

Dr. N. Radhakrishnan, Chief of the Automation Technology Center (ATC), WES, SSI Studies Project Manager, and Dr. Robert L. Hall, Research Civil Engineer, WES, ATC, coordinated and monitored the work. WES Publications and Graphic Arts Division Editor, Gilda Shurden, supplied editorial support. Mr. Donald R. Dressler, Structures Division, Civil Works Directorate, was the OCE point of contact.

Director of WES was COL Allen F. Grum, USA. Technical Director was Dr. Robert W. Whalin.





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CONVERSION FACTORS, NON-SI TO SI (METRIC) UNITS OF MEASUREMENT

U.S. customary (non-SI) units of measurement used in this report can be converted to metric (SI) units as follows:

Multiply	Ву	To Obtain
feet	3.048	metres
inches	2.54	centimetres
kip (force)-ft	1355.818	newton-metres
kips (force)	4.448222	kilonewtons
kips (force) per square inch	6.894757	megapascals
kip-inches	112.9848	newton-metres

FOUNDATION INTERACTION PROBLEMS INVOLVING AN ELASTIC HALF-PLANE

PART I: INTRODUCTION

Background

1. Stress analysis problems involving interaction between a structure and its foundation often lead to extensive computational effort. This fact becomes obvious when finite-element methods are used to study problems where the foundation is idealized as an infinite elastic half-plane. Attempts to represent the half-plane by a finite size structure using many elements entail solving large systems of simultaneous equations at a considerable computational cost.

Purpose

- 2. This report presents a more computationally efficient procedure which employs the exact solution of the equations of elasticity for an elastic half-plane subjected to arbitrary surface loading. Complex variable formulations are shown to yield a compact solution for the stresses and displacements in a half-plane supporting several concentrated loads.
- 3. This solution is also employed to compute flexibility and stiffness matrices relating the concentrated loads and the displacements at the points of application of the loads. The stiffness matrix derived in this manner is then employed to investigate a simple type of interaction problem where an Euler beam rests on an elastic half-plane and is subjected to external loading. The foundation interaction forces, as well as other quantities such as beam shear and moment, are also computed.

PART II: TWO-DIMENSIONAL ELASTOSTATIC PROBLEMS

Significant Effects

4. A brief summary of the various field quantities important in the two-dimensional infinitesimal deformation theory of linear elasticity is presented below. The stress state is represented by two extensional stresses τ_{xx} and τ_{yy} and a shear stress τ_{xy} .* Hooke's law states that the stresses depend linearly on the extensional strains ε_{xx} and ε_{yy} and the shear strain ε_{xy} . Furthermore, the strains are functions of the first derivatives of the displacements u (in the x-direction) and v (in the y-direction). The governing differential relations are

$$\varepsilon_{xx} = \frac{\partial u}{\partial x}$$
, $\varepsilon_{yy} = \frac{\partial v}{\partial y}$, $\varepsilon_{xy} = \frac{1}{2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)$

5. Hook's law for a homogeneous, isotrophic material contains three elastic constants which are Young's modulus, E, Poisson's ratio, σ , and the shear modulus, G. Only two of these are independent because

$$G = \frac{E}{[2(1+\sigma)]}$$

Since a two-dimensional elasticity problem is a special case of a more general three-dimensional configuration, it is instructive to examine the restriction involved in the two-dimensional specialization. In three dimensions we have to consider the following quantities, each of which depends on spatial coordinates \mathbf{x} , \mathbf{y} , and \mathbf{z} .

- a. Three displacements u, v, w
- $\frac{b}{z}$. Six stresses $\frac{\tau}{xx}$, $\frac{\tau}{vy}$, $\frac{\tau}{zz}$, $\frac{\tau}{xy}$, $\frac{\tau}{vz}$, $\frac{\tau}{zx}$
- \underline{c} . Six strains ε_{xx} , ε_{yy} , ε_{zz} , ε_{xy} , ε_{yz} , ε_{zx}

S. P. Timoshenko and J. N. Goodier. 1970. Theory of Elasticity, 3d ed., McGraw-Hill, New York.

Subclasses--Plane Strain and Stress

6. The main subclasses of two-dimensional problems, plane strain and plane stress, are mathematically similar. A state of plane strain exists when the displacement \mathbf{w} (in the z-direction) vanishes and the displacements \mathbf{u} and \mathbf{v} are functions of \mathbf{x} and \mathbf{y} only. It is then found that for plane strain

$$\varepsilon_{yz} = \varepsilon_{zx} = \varepsilon_{zz} = 0$$

$$\tau_{yz} = \tau_{zx} = 0, \quad \tau_{zz} = \sigma(\tau_{xx} + \tau_{yy})$$

the stresses $\frac{\tau}{xx}$, $\frac{\tau}{yy}$, and $\frac{\tau}{xy}$ related to strains $\frac{\varepsilon}{xx}$, $\frac{\varepsilon}{yy}$, and $\frac{\varepsilon}{xy}$ seconding to

$$2G \stackrel{\varepsilon}{\approx}_{xx} = (1 - \sigma)\tau_{xx} - \tau_{yy}, \quad 2G \stackrel{\varepsilon}{\approx}_{yy} = (1 - \sigma)\tau_{yy} - \tau_{xx}$$

$$2G \stackrel{\varepsilon}{\approx}_{xy} = \tau_{xy}$$

tress state referred to as plane stress occurs when all field quantities we independent of ${\bf z}$ and

$$\tau_{xz} = \tau_{yz} = 0$$
, $\tau_{zz} = 0$

$$\epsilon_{xz} = \epsilon_{yz} = 0$$
, $\epsilon_{zz} = \frac{-\sigma(\tau_{xx} + \tau_{yy})}{E}$

remaining nonzero field quantities $\frac{\tau}{xx}$, $\frac{\tau}{yy}$, $\frac{\tau}{xy}$, $\frac{\varepsilon}{xx}$, $\frac{\varepsilon}{yy}$, and are related for the case of plane stress by

$$2G \in \frac{\tau}{xx} = \frac{\tau - \sigma \tau}{1 + \sigma}$$

$$2G \quad \varepsilon_{yy} = \frac{\tau_{yy} - \sigma \tau_{xx}}{1 + \sigma}$$

$$2G \quad \varepsilon_{xy} = \tau_{xy}$$

7. Because the equations arising in plane strain and plane stress are very similar, they can be solved by the same mathematical procedures. It is convenient to use as fundamental elastic constants the shear modulus G and another parameter κ where

$$\kappa = 3 - 4\sigma$$

for plane strain and

$$\kappa = \frac{3 - \sigma}{1 + \sigma}$$

for plane stress. Then the stress-strain formulas for both situations turn out to be

$$2G \epsilon_{xx} = \frac{\tau_{xx} + (\kappa - 3)(\tau_{xx} + \tau_{yy})}{4}$$

$$2G \ \epsilon_{yy} = \frac{\tau_{yy} + (\kappa - 3)(\tau_{xx} + \tau_{yy})}{4}$$

$$2G \epsilon_{xy} = \tau_{xy}$$

Although the constant \times does not have an obvious physical significance comparable to that of σ , it is, nevertheless, much more convenient to use in a variety of formulas needed in the work which follows.

Boundary Value Problems

8. Solving a boundary value problem in plane elasticity entails determining the stresses τ_{xx} , τ_{yy} , and τ_{xy} and the displacements u and v that result from given boundary conditions. The first fundamental type problem involves the case where surface traction components T_x and T_y are given. These tractions depend on the boundary values of stress according to

$$T_x = \tau_{xx}^{\vee} + \tau_{xy}^{\vee}$$

$$T_y = \tau_{xy}^{\vee} + \tau_{yy}^{\vee}$$

with ν_x and ν_y being the x and y components of the outward directed unit surface normal. Assuming that the boundary tractions comply to self-equilibrating loads, the stresses throughout the body are determined uniquely and the displacements are determined within a rigid body displacement of the form

$$u = u_0 - \alpha_0 y$$
, $v = v_0 + \alpha_0 x$

where \mathbf{u}_0 and \mathbf{v}_0 are translation components and α_0 is an angle of rotation. The arbitrary parameters \mathbf{u}_0 , \mathbf{v}_0 , and α_0 can be fixed by specifying the displacement of a selected point and the direction of one line segment through the point.

9. Two other types of problems also merit interest. In the second fundamental type boundary value problem, displacements are known overall on the boundary. Then, the stresses and displacements will be uniquely determined throughout the body. A third, and more difficult, type problem involves so-called mixed boundary conditions where tractions are known on one boundary part and displacement conditions are known on another. The mixed problem is not studied here extensively. However, one important mixed problem which is dealt with to some extent involves a half-plane having zero load on one part with displacement conditions relating to an Euler beam supported by the half-plane. To analyze this problem, developments are presented in the following paragraphs relating to a half-plane under general loading and an Euler beam under general loadings.

Complex Variable Formulation

10. The stresses and displacements in a two-dimensional linear elastostatic problem can be expressed in terms of two analytic functions $\Phi(z)$ and f(z) and their integrals*,**

^{*} Timoshenko and Goodier, op. cit.

^{**} N. I. Muskhelishvili. 1953. <u>Some Basic Problems of the Mathematical Theory of Elasticity</u>, Noordhoff, Groningen.

$$\phi(z) = \int \Phi(z) dz$$
 and $\psi(z) = \int \Psi(z) dz$

When the boundary conditions and the boundary geometry have simple enough form, general solutions for ϕ and ψ can be written. Included in this category are problems for the circle and the half-plane. Several stress formulas needed here are summarized below. These and other such relations are developed in great detail in the reference work by N. I. Muskhelishvili.*

II. The stresses and displacements are related to $\, \varphi$, $\, \Psi$, $\, \varphi$, and $\, \psi$, according to Kolosov's formulas which are

$$\tau_{xx} + \tau_{yy} = 2\left[\phi(z) + \overline{\phi(z)}\right]$$

$$-\tau_{xx} + \tau_{yy} + 2i\tau_{xy} = 2[\bar{z}\phi'(z) + \Psi(z)]$$

$$2G(u + iv) = \kappa \phi(z) - z \overline{\phi(z)} - \overline{\psi(z)}$$

where

$$i = \sqrt{-1}$$
 , $z = x + iy$

and

$$\kappa = 3 - 4\sigma$$

for plane strain or

$$\kappa = \frac{3 - \sigma}{1 + \sigma}$$

for plane stress.

12. In a problem leading to a unique stress state, then ϕ is determined uniquely except for an additive pure imaginary constant, and Ψ is

^{*} Muskhelishvili, op. cit.

completely determined. Thus, if the function pairs (Φ, Ψ) and (Φ_0, Ψ_0) give the same stresses, we must have

$$\Phi_0 = \Phi + ic_1$$
, $\Psi_0 = \Psi$

where c_1 is real. Integrating for ϕ and Ψ shows that

$$\phi = \phi + ic_1z + c_2, \quad \psi_0 = \psi + c_3$$

with c_2 and c_3 being complex. The constants, c_1 , c_2 , and c_3 , correspond to a rigid body displacement field of the form

$$2G(u + iv) = (\kappa + 1)c_1(-y + ix) + (\kappa c_2 - c_3)$$

In a problem where only stresses are unique, unique displacements can be obtained by specifying a displacement and rotation at one point. Then both ϕ and ψ become unique.

13. The above formulas can be used to reduce the solution of plane elasticity problems to an equivalent formulation requiring determination of $\Phi(z)$ and $\Psi(z)$. Furthermore, these functions can be computed in general form for certain restricted types of geometries and boundary conditions. A case of special interest here involves the elastic half-plane subjected to stress-type boundary conditions. Let us assume that a half-plane occupies the region $-\infty < x < \infty$ and $y \le 0$. The line y = 0 divides the plane into two parts:

the region R^+ defined by y > 0 and the region R^- defined by y < 0

The stress and displacement quantities of interest will exist in R^- . However, some of the mathematical formulas presented below contain quantities defined for both R^+ and R^- with appropriate limiting values being used for approaches to the boundary y=0 from above or below.

14. Consider the case with the normal and shear stresses known at all points of the boundary. Thus, we have

$$\tau_{yy}^-(t) = N(t) \quad \tau_{xy}^-(t) = T(t)$$

where N and T are known functions. The complex stress functions which solve this problem are

$$\Phi(z) = -\frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{(N - iT) dt}{t - z}$$

and

$$\Psi(z) = -\frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{(N + iT) dt}{t - z} - \Phi(z) - z\Phi'(z)$$

Furthermore, the boundary stresses and displacements are related to the boundary values of Φ in a remarkably simple form according to

$$N + iT = \Phi^+(t) - \Phi^-(t)$$

where $\phi^+(t)$ and $\phi^-(t)$ denote limiting values of $\phi(z)$ at a boundary point t approached from above or below the x-axis, respectively. The theoretical developments relating to these formulas appear in Chapters 16 and 19 of Muskhelishvili's* elasticity book.

^{*} Muskhelishvili, op. cit.

PART III: FUNDAMENTAL LOAD FORMULAS FOR THE HALF-PLANE

Stress-Function Methodology

15. The complex stress-function methodology leads to simple relations for the stresses and displacements in a half-plane subjected to certain fundamental loading conditions such as application of a load distributed over a uniform strip. These solutions can be employed to develop the stiffness and flexibility matrices relating stresses and displacements that occur when a series of vertical loads, horizontal loads, and couples are applied to the surface of a half-plane.

Vertical and Horizontal Load Solutions

16. Let us first develop a solution for the case of a vertical load P_0 and a horizontal load V_0 , distributed uniformly over the strip -a < x < a. Thus, we have

$$N + iT = 0$$
 for $|t| > a$

and

$$N + iT = \frac{P_0 + iV_0}{2a}$$
 for $|t| < a$

Then

$$\phi(z) = \frac{V_0 + iP_0}{4\pi a} [\ln(z - a) - \ln(z + a)]$$

where $\phi(z)$ is an analytical function in the plane cut along a straight line between z=-a and z=+a. The formula for ϕ can be used to compute the boundary displacements which are uniquely determined except for a rigid body displacement and rotation.

17. The boundary values of ϕ are given by

$$\Phi^{\pm}(t) = \gamma_0 \left[\ln |t - a| - \ln |t + a| \pm i\theta(t) \right]$$

where

$$\gamma_0 = \frac{V_0 + iP_0}{4\pi a}$$

and

$$\theta(t) = 0$$
 for $|t| > a$ and $\theta(t) = \pi$ for $|t| < a$

Consequently,

$$2\mu \Big[u'(t) + iv'(t) \Big] = (\kappa + 1) \gamma_0 \Big[\ln |t - a| - \ln |t + a| \Big] + i(\kappa - 1) \gamma_0^{\pi}$$

for t < a

and

$$2\mu \left[u'(t) + iv'(t) \right] = (\kappa + 1) \gamma_0 \left[\ln \left| t - a \right| - \ln \left| t + a \right| \right] \quad \text{for} \quad |t| > a$$

18. The last two equations can be combined by using the singularity function ${\langle t-t_0 \rangle}^n$ which equals ${(t-t_0)}^n$ when $t > t_0$ and is zero otherwise. Then we get

$$u'(t) + iv'(t) = \alpha_0 r(t) + i\beta_0 \left[\langle t + a \rangle^0 - \langle t - a \rangle^0 \right]$$

where

$$\alpha_0 = \frac{(\kappa + 1)(v_0 + iP_0)}{8\mu\pi a}$$

$$\beta_0 = \frac{(\kappa - 1)(v_0 + iP_0)}{2\mu_a}$$

and

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$$r(t) = \ln|t - a| - \ln|t + a|$$

Expressions for u(x) + iv(x) can be obtained by integrating u' + iv'. It is not hard to show that

$$\int_{0}^{x} \left[\langle t + a \rangle^{0} - \langle t - a \rangle^{0} \right] dt = \langle x + a \rangle^{1} - \langle x - a \rangle^{1} - a = g(x)$$

and

$$\int_{0}^{x} \left[\ln |t - a| - \ln |t + a| \right] dt = (x - a) \ln |x - a| - (x + a) \ln |x + a| + 2a \ln (a) = f(x)$$

so the general displacements equation for a distributed load is

$$u(x) + iv(x) = \alpha_0 f(x) + i\beta_0 g(x)$$

The function g(x) is odd valued and has three different intervals of definition, namely

$$g(x) = -a$$
, $x < -a$
= x , $-a \le x \le a$
= a , $x > a$

Furthermore, f(x) is an even-valued function having a logarithmic singularity at $|x| = \infty$. These two functions form the basis for computing the deformation effects of vertical or horizontal forces as well as couples. Several special cases arise.

<u>a.</u> Case I--Vertical Force Only (positive direction upward). In this instance, we have

$$v_0 = 0$$

so that

$$\alpha_0 = \frac{iP_0(1 + \kappa)}{8\mu\pi a}$$

and

$$\beta_0 = \frac{iP_0(\kappa - 1)}{8\mu a}$$

Then

$$u(x) + iv(x) = i \left[\frac{P_0(1 + \kappa)}{8\mu\pi a} \right] f(x) - \left[\frac{P_0(\kappa - 1)}{8\mu a} \right] g(x)$$

The real and imaginary parts of this equation give

$$v(x) = \frac{P_0(1 + \kappa)}{8\mu\pi a} [f(x) - f(e)]$$

and

$$u(x) = \frac{-P_0(\kappa - 1)}{8ua} [g(x)]$$

where a rigid body translation term has been added into the $\mathbf{v}(\mathbf{x})$ equation to make $\mathbf{v}=0$ at an arbitrary reference point $\mathbf{x}=\mathbf{e}$.

<u>b.</u> Case II--Horizontal Force Only (positive direction to the right). In this instance $P_0 = 0$, so

$$\alpha_0 = \frac{(\kappa + 1)V_0}{8\mu\pi}$$
 and $\beta_0 = \frac{(\kappa - 1)V_0}{8\mu a}$

Then

$$u(x) = \frac{V_0(\kappa + 1)}{8u\pi a} [f(x) - f(e)]$$

$$v(x) = \frac{v_0(\kappa - 1)}{8\mu a} [g(x)]$$

where an adjustment has been made in the horizontal displacement equation to make u vanish at x = e.

c. Case III--Couple (positive direction counterclockwise). The load effect of a couple can be obtained by combining the effects of two forces of equal magnitude and opposite direction. Let us represent a couple of magnitude M_0 by placing a force $-M_0/\Delta$ at x=0 and a force M_0/Δ at $x=\Delta$. We then let Δ approach zero. The resulting displacement equation is

$$u(x) = \frac{M_0(\kappa + 1)}{8\mu\pi a} \lim_{\Delta \to 0} [f(x - \Delta) - f(x)]/\Delta$$

which becomes

$$u(x) = \frac{-M_0(\kappa \pi)}{8u\pi a} [f'(x)] = \frac{-M_0(\kappa + 1)}{8u\pi a} [r(x)]$$

and similarly,

$$v(x) = \frac{M_0(\kappa - 1)}{8\mu a} [g'(x)] = \frac{M_0(\kappa - 1)}{8\mu a} [\langle x + a \rangle^0 - \langle x - a \rangle^0]$$

Flexibility and Stiffness Matrices

19. In order to form the flexibility and stiffness matrices relating loads and displacements for a series of vertical forces, horizontal forces, and couples, it is necessary to know the deflection equations and also the rotation equations for the three different types of load quantities. It is not hard to see that the rotation of any initially horizontal element on the surface of the half-plane is

$$\theta(x) = \frac{dv(x)}{dx}$$

so the rotation function is obtained by differentiating the vertical displacement equation.

20. The purpose of distributing forces over a width 2a is to avoid displacement singularities occurring at the point where a concentrated load is applied. In a practical application, the value used for the width parameter should be small compared to the distance between successive load application points. Taking a = d/20, where d is the smallest distance between adjacent load points, seems to be a reasonable choice. It must be realized that results obtained will depend, somewhat, on the choice of parameter a. The size of a should be small enough so that a concentrated load is satisfactorily replaced by a distributed load. At the same time, this parameter must be large enough so that round off errors do not cause inaccurate calculation of the various influence functions.

Displacement Loads

21. Let us now summarize the various displacement formulas corresponding to unit-load quantities applied at x = 0. These relations involve elastic constants

$$c_0 = \frac{\kappa + 1}{8\mu\pi a}$$
 and $e_0 = \frac{\kappa - 1}{8\mu a}$

and functions

$$f(x) = (x - a) \ln |x - a| - (x + a) \ln |x + a|$$

$$r(x) = f'(x) = \ln |x - a| - \ln |x + a|$$

$$s(x) = f''(x) = \frac{1}{(x - a)} - \frac{1}{(x + a)} = \frac{2a}{(x^2 - a^2)}$$

$$g(x) = \langle x + a \rangle^1 - \langle x - a \rangle^1 - a$$

$$h(x) = g'(x) = \langle x + a \rangle^0 - \langle x - a \rangle^0$$

where f , s , and m are even valued and r and g are odd valued. 22. For a vertical load at x = 0 we have

$$v = c_0[f(x) - f(e)] = c_0F(x)$$

$$u = -e_0g(x)$$

$$\theta = c_0r(x)$$

For a horizontal load at x = 0 we have

$$v = e_0 g(x)$$

$$u = c_0 F(x)$$

$$\theta = e_0 h(x)$$

For a couple at x = 0 we have

$$v = -c_0 r(x)$$

$$u = e_0 h(x)$$

$$\theta = -e_0 s(x)$$

23. These functions can be used to formulate the flexibility matrix corresponding to the circumstance where horizontal forces, vertical forces, and couples are applied at a series of points on the surface of a half-plane. A program is given below which forms a 3n by 3n flexibility matrix for a general set of points \mathbf{x}_1 ,..., \mathbf{x}_n . Horizontal, vertical, and rotational quantities in the matrix correspond to row and column positions 3i-2, 3i-1, and 3i for i=1,..., n.

PART IV: ALTERNATIVE REAL-VARIABLE FORMULATION RESTRICTED TO VERTICAL LOADS

Complex Variable Methods

- 24. Complex variable methods were employed above to derive various results applicable for a half-plane. When only vertical loads and deflections are of interest, then equivalent results can be obtained more simply by using a real-variable formulation. This specialized loading condition also has a meaningful interpretation in a three-dimensional context where a finite-width beam rests on a half-space and is subjected to loads symmetrical about the vertical midplane through the longitudinal axis of the beam. The vertical-loading problem will now be studied in more detail before moving on to analysis of the beam-interaction problem.
- 25. Consider an infinite half-space the surface of which is the xz plane and the interior of which is defined by y < 0. We assume that concentrated loads P_i in the y direction are applied at positions x_i , i = 1,..., n on the x-axis. It is desired to compute the foundation stiffness matrix K_F in the relation

$$P = K_F \tilde{V}$$

which involves the applied loads $\,P$, the vertical deflections $\,^{\,\!\!\!V}$, and the stiffness matrix $\,^{\,\!\!\!K}_F$. Formulation of the stiffness matrix is to be based upon the linear theory of elasticity. The plane analog of this problem occurs when the concentrated point loads are replaced by concentrated line loads having a given magnitude per unit of thickness in the z-direction. Each of these fundamental problems is discussed below.

26. The deflection functions associated with a concentrated load applied to the surface of a half-space are well known. When a

^{*} Timoshenko and Goodier, op. cit. p 5.

^{**} Muskhelishvili, op. cit. p 8.

[†] C. S. Desai and J. T. Christian. 1977. <u>Numerical Methods in Geotechnical Engineering</u>, McGraw-Hill, New York.

^{††} A. P. S. Selvadurai. 1979. Elastic Analysis of Soil Foundation Interaction, Elsevier, New York.

vertical force P_0 is applied at some surface position x_i , then the vertical deflection occurring at another surface position x_i is given by

$$v_{ij} = -\frac{(1 - \sigma^2)P_0}{\pi E_0 |x_i - x_j|}$$

where $\rm E_0$ and σ denote Young's modulus and Poisson's ratio. A similar solution exists when the concentrated point load is replaced by a concentrated line load distributed parallel to the z-axis. In that instance the corresponding formulas relating load and deflection are

$$v_{ij} = -\frac{2(1 - \sigma^2)P_0}{\pi E_0} \ln |x_i - x_j| \quad \text{for plane strain}$$

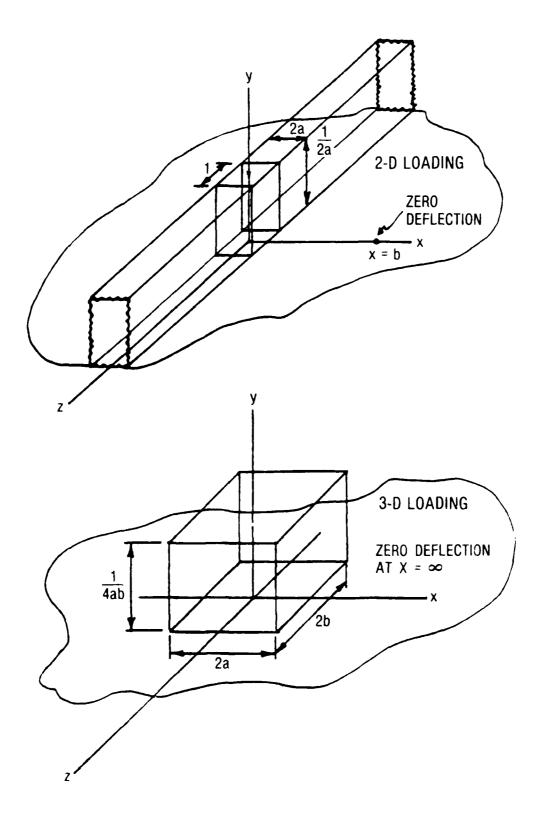
and

$$v_{ij} = -\frac{2P_0}{\pi E_0} \ln |x_i - x_j|$$
 for plane stress

Infinite Displacements

27. These formulas share the same undesirable characteristic of giving infinite displacements at the point of load application. This difficulty can be remedied by averaging the load over a finite area as shown in Figure 1. Consider the case where a concentrated load P_0 is distributed uniformly over the region $-a \le x \le a$, $-b \le z \le b$. Then the deflection at any position on the x-axis is seen by superposition to be

$$v = \frac{P_0(1 - \sigma^2)}{4\pi E_0 ab} \int_{-a}^{a} \int_{-b}^{b} \frac{dr_0 d\xi}{\sqrt{r_0^2 + (\xi - x)^2}}$$



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Figure 1. Two- and three-dimensional loading diagrams

or

$$v = \frac{P_0(1 - \sigma^2)}{2\pi E_0 ab} \int_{x-a}^{x+a} \int_{0}^{b} \frac{dn \ d\xi}{\sqrt{n^2 + \xi^2}}$$

Evaluation of the above double integral is tedious but can be accomplished in terms of elementary functions. It turns out that

$$v(x) = \frac{P_0(1 - \sigma^2)}{\pi E_0 a} \left\{ \frac{(a - x)}{b} \ell_n \left[\frac{b + \sqrt{b^2 + (a - x)^2}}{|a - x|} \right] + \frac{(a + x)}{b} \ell_n \left[\frac{b + \sqrt{b^2 + (a + x)^2}}{a + x} \right] + \ell_n \left[\frac{x + a + \sqrt{b^2 + (x + a)^2}}{x - a + \sqrt{b^2 + (a - x)^2}} \right] \right\}$$

In the above formula x is regarded as a positive quantity. When x is negative, |x| should be used. It can be shown that as x becomes large, the last formula assumes the asymptotic form

$$v(x) \cong \frac{P_0(1-\sigma^2)}{\pi E_0} \frac{1}{x}$$

as would be expected from the concentrated load solution. In fact, when x/a is larger than six, the difference between the concentrated load solution and the distributed load solution is negligible regardless of the value of b.

28. A distributed load solution can also be obtained for the case of plane stress or plane strain. Let us assume that a load of P_0 per unit of z-thickness is distributed over the region -a < x < a. Using superposition for the plane strain solution yields

$$v(x) = \frac{P_0(1-\sigma^2)}{{}^{\pi}E_0a} \int_{-a}^{a} \ell_n |_{x} - \xi |_{d\xi} = -\frac{P_0(1-\sigma^2)}{{}^{\pi}E_0a} \int_{x-a}^{x+a} \ell_n |\xi|_{d\xi}$$

which gives

$$v(x) = -\frac{P_0(1-\sigma^2)}{\pi E_0 a} \left[\xi \ln |\xi| - \xi \right] \frac{x+a}{x-a} = \frac{P_0(1-\sigma^2)}{\pi E_0 a} \left[f(x) - 2a \ln(a) + 2a \right]$$

where f(x) is the same function obtained earlier in the development using complex variables. Hence, the displacement formulas given by the two methods are the same with the exception of a rigid body translation. The analogous formula for plane stress results simply by replacing

$$(1-\sigma^2)$$
 with 1

29. The distributed load solution for the half-plane is bound at x=0 but has a logarithmic singularity at $x=\infty$. As a remedy for this, it is reasonable to choose a value e which is much larger than a and perform a rigid body translation such that v=0 at $x=\pm e$. Thus, we assume that the deflection pattern for plane strain and a concentrated line load is

$$v(x) = \frac{P_0(1 - \sigma^2)}{\pi E_0 a} [f(x) - f(e)]$$

where

$$f(x) = -(x + a)\ln|x + a| + (x - a)\ln|x - a|$$

30. In the instance of both plane loading and three-dimensional loading, the formulas for deflection at a distance x from the point of load application are of the form

$$v(x) = c_0 F(x, a, b)$$

where F does not depend on the elastic constants. The constant c_0 is inversely proportional to aE_0 and is influenced by Poisson's ratio only in the cases of plane strain and three-dimensional loading. By superposition,

the surface deflection at any position x due to loads P_{i} at x_{i} is

$$v(x) = c_0 \sum_{j=1}^{n} F(x - x_j, a, b)P_j$$

Evaluating this relation at x_i to get v_i gives

$$v_{i} = c_{0} \sum_{j=1}^{n} h_{ij} P_{j}$$

or

$$\tilde{V} = c_0 HP$$

where the foundation flexibility matrix H has elements

$$h_{ij} = F(x_i - x_j, a, b)$$

Inverting this relation yields the result

$$P = K_F \tilde{V}$$

where the desired foundation stiffness matrix $K_{\overline{F}}$ is related to the flexibility coefficients according to

$$K_{\mathbf{F}} = \frac{1}{c_0} \, \mathbf{H}^{-1}$$

It is important to keep in mind that H depends only on dimension parameters a , b , and \mathbf{x}_i , whereas \mathbf{c}_0 involves \mathbf{E}_0 , $(1-\sigma^2)$, and parameter a , with the functional dependence which these variables have on \mathbf{K}_F being quite simple in form.

PART V: ANALYSIS OF AN EULER BEAM HAVING DISTRIBUTED AND CONCENTRATED LOADS

Solution for Load Deflection Relations

- 31. Before an analysis can be made of the interaction between a beam and a half-plane, necessary load deflection relations for a beam are needed. Toward this objective, a concise solution is presented below for the shear, moment, slope, and deflection in an Euler beam having constant depth and subjected to an arbitrary combination of concentrated loads and piecewise linearly varying (ramp) loads. This solution will be used to investigate the interaction between a loaded beam and the supporting elastic half-plane. The contact between the beam and the half-plane occurs at several support points which can transmit only vertical concentrated forces. Displacement continuity conditions between the beam and the plane are imposed at the support points. By using the displacement formulas for a beam and a half-plane subjected to the same concentrated interaction loads, imposition of displacement continuity conditions at the interaction points gives a system of simultaneous equations solvable for the support forces.
- 32. Beam deflection problems can be conveniently formulated by using the singularity function $(x x_0)^n$ defined such that

$$(x - x_0)^n = 0$$
 for $x < x_0$
= $(x - x_0)^n$ for $x \ge x_0$

where n is a nonnegative integer. This type of function is quite useful for describing general loading conditions.

33. The analysis of beam deflection problems involves the quantities: load per unit length = w

shear = V

moment = M

slope = s

vertical deflection = v

The differential equations relating these variables are

$$\frac{dV}{dx} = w$$
, $\frac{dM}{dx} = V$, $\frac{ds}{dx} = \frac{M}{EI}$, $\frac{dv}{dx} = s$

where

E = Young's modulus

I = moment of inertia of the beam

Fundamental Solutions

34. It is convenient to introduce several fundamental solutions used in forming more general solutions by superposition. Consider first the effects of a unit load. Assume that a beam occupies the region $x \ge 0$ and satisfies, at x = 0, the homogeneous conditions V = 0, M = 0, S = 0, and V = 0. Also assume that a concentrated load of unit magnitude acts at X = 0. It is found that

$$V = {\langle x - c \rangle}^{0}$$
, $M = {\langle x - c \rangle}^{1}$
 $s = \frac{1}{2} \frac{{\langle x - c \rangle}^{2}}{EI}$, $v = \frac{1}{6} \frac{{\langle x - c \rangle}^{3}}{EI}$

35. The load and deformation quantities associated with ramp loading are also of interest. Assume that a distributed load begins at x = a with magnitude R and varies linearly to x = b where the load intensity is T. The slope of the load function is S = (T - R)/(b - a) and the load function for this case is

$$w = R <_X - a>^0 + S<_X - a>^1 - T<_X - b>^0 - S<_X - b>^1$$

Integration of the differential equation, subject to homogeneous initial conditions, gives

$$V = R < x - a > {}^{1} + \frac{1}{2} S < x - a > {}^{2} - T < x - b > {}^{1} - \frac{1}{2} S < x - b > {}^{2}$$

$$M = \frac{1}{2} R < x - a > {}^{2} + \frac{1}{6} S < x - a > {}^{3} - \frac{1}{2} T < x - b > {}^{2} - \frac{1}{6} S < x - b > {}^{3}$$

$$EIs = \frac{1}{6} R < x - a > {}^{3} + \frac{1}{24} S < x - a > {}^{4} - \frac{1}{6} T < x - b > {}^{3} - \frac{1}{24} S < x - b > {}^{4}$$

EIv =
$$\frac{1}{24}$$
 R4 + $\frac{1}{120}$ S5 - $\frac{1}{24}$ T4 - $\frac{1}{120}$ S5

36. It is helpful to represent the previous equations in the following compact form. We designate

FUNCTION FUNIT (X,C,EI,ID)

as the function which gives the effects of a unit concentrated load. The function produces shear, moment, slope, or deflection according to whether ID = 1, 2, 3, or 4, respectively. Similarly, we designate

FUNCTION RAMP(X,A,R,B,T,EI,ID)

as the function returning the effects of a ramp loading.

- 37. The fundamental solutions shown above can be used to formulate and solve a general problem involving multiple loads. Assume that the following apply:
 - a. At the left end where x = 0, the moment and shear vanish.
 - \underline{b} . At the right end where $x = \ell$, the deflection and the slope vanish.
 - \underline{c} . At several points external concentrated loads act. These are described by

$$F_{i}$$
 at d_{i} , $j = 1$, ..., nf

- \underline{d} . At several points foundation reactions occur. These are described by P_j at x_j , j=1, ..., n. The foundation reactions are initially unknown but are treated as parameters to be computed later using displacement constraints.
- Along several intervals distributed loads act. These are described by

$$w_{j}(x) = R_{j}^{<}x - a_{j}^{>0} + S_{j}^{<}x - a_{j}^{>1}$$

$$- T_{j}^{<}x - b_{j}^{>0} - S_{j}^{<}x - b_{j}^{>1}, \quad j = 1, \dots, nr$$

38. Using the various terms developed above, the deflection equation for a beam with general loading and general end conditions is

$$v_B(x) = v_B(0) + v_B'(0)x + v_{ext}(x) + \sum_{j=1}^{n} P_j *FUNIT(x, x_j, EI, 4)$$

where $v_B^{}(0)$ and $v_B^{}(0)$ represent the left end deflection and slope which are for the present not known and

$$v_{ext}(x) = \sum_{j=1}^{nf} F_j * FUNIT(x, d_j, EI, 4)$$
 $+ \sum_{j=1}^{nr} RAMP(x, a_j, R_j, b_j, T_j, EI, 4)$

PART VI: COMBINED LOAD DEFLECTION RELATIONS FOR THE BEAM AND THE FOUNDATION

Analysis Procedures

- 39. The analysis developed in Part V provides the tools needed to form combined flexibility matrices for computing interaction forces between a loaded beam and a supporting half-plane. The necessary procedure is described in the following paragraphs.
- 40. The beam is subjected to a general external loading w(x) plus a series of reactions P_i applied at the foundation support points x_i for i=1, ..., n. The boundary conditions on the beam are that the shear and moment vanish at each end. Integrating the differential equation for the beam deflection gives an equation of the form

$$v_B(x) = v_B(0) + v_B'(0)x + v_{EXT}(x) + \sum_{j=1}^{n} h_j(x)P_j$$

where subscript B distinguishes the beam and P_1 , ..., P_n , $v_B(0)$, $v_B'(0)$ are to be determined. The function $v_{EXT}(x)$ represents the combined deflection contribution of all loads except for the foundation reactions. The function

$$h_1(x) = \frac{(x - x_1)^3}{6FI}$$

represents the deflection contribution for a unit load applied at \mathbf{x}_j . The conditions of zero shear and moment at the right end are

$$\sum_{j=1}^{n} P_{j} = -V_{EXT}(\ell)$$

and

$$\sum_{j=1}^{n} (\ell - x_j) P_j = -M_{EXT}(\ell)$$

where $\mathbf{V}_{\mathbf{EXT}}$ and $\mathbf{M}_{\mathbf{EXT}}$ refer to shear and moment terms caused by the external loads.

Singularity Avoided

41. Considering the foundation, let t(x) represent the surface deflection at position x resulting from a unit load applied at x=0. To avoid the singularity in the concentrated load solution for a half-plane, it is understood that the concentrated load is approximated by a statically equivalent uniform pressure applied over a narrow strip. This strip is considered narrow, having a width small compared to the total length of the beam. A strip width of $\ell/1000$ might be reasonable. Using superposition indicates that the foundation surface deflection caused by surface loads P_1 , ..., P_n is

$$v_{F}(x) = -\sum_{j=1}^{j=n} t(x - x_{j})P_{j}$$

Therefore, the flexibility matrix relating surface loads and deflections and displacements is

$$v_{\mathbf{F}}(\mathbf{x_i}) = -\sum_{\mathbf{j=1}}^{n} t_{\mathbf{ij}}^{\mathbf{P_j}}$$

involving a symmetric matrix with elements $t_{ij} = t(x_i - x_j)$. A negative sign in the last two equations is used because an upward reaction on the beam would correspond to a downward (negative) displacement in the foundation. Matching displacements in the beam and half-plane at reaction points given

$$\sum_{j=1}^{n} [h_{ij} + r_{ij}] P_{j} + v_{B}(0) + v_{B}'(0) x_{i} = -v_{EXT}(x_{i}), \quad i = 1, 2, ..., n$$

where

$$h_{ij} = h_{j}(x_{i})$$

is supplemented with the conditions of vanishing shear and moment at the right end, a system of n+2 equations is obtained which is solvable for P_1 , P_2 ,..., P_n , $v_B(0)$, $v_B^{\dagger}(0)$.

42. Assuming that the foundation and beam load transfer mechanism takes place at a finite number of support points amounts to an effort to discretize a continuous foundation pressure.

Foundation Pressure Discretization

- 43. It was assumed that load transfer and displacement continuity occur only at a finite number of support points along the beam. In this manner, a discretization of the continuously varying foundation pressure is achieved. For satisfactory results, it is probably necessary to use 40 or more interaction points. After the concentrated load reactions have been found, a statically equivalent piecewise linear load distribution can be derived which better represents the actual interface pressure. With all loads on the beam being known, any additional quantities, such as internal shear and moment, can be computed for arbitrary positions on the beam.
- 44. The procedure just developed employs flexibility matrices. In some cases it may be more convenient to use a stiffness matrix formulation. Such a formulation outline follows. Consider a beam with the right end free and left end values of shear, moment, slope, and deflection which are F_0 , $-M_0$, v'(0), and v(0). By the previously discussed methods for a beam, it is found that the deflection v_i at position x_i associated with loads R_1 , ..., P_n is

$$v_i = v(0) + v'(0)x_i + \sum_{j=1}^n b_{ij}^p$$

where

$$b_{ij} = b_{ji} = \frac{\left[x_i^2 (3x_j - x_i) + \langle x_i - x_j \rangle^3\right]}{6EI}$$

Using matrix notation we can write

$$[v_1, ..., v_n]^T = \tilde{v}, [v(0), v'(0)]^T = V_0$$

and

Assume that the vector of nodal forces $\{P_1,\ldots,P_n\}^T=P$ be composed of external loads P_{EXT} and foundation reactions P_F . In matrix form, the beam deflection equation gives $\tilde{V}=RV_0+BP$ which also implies that $P=K_B\tilde{V}-K_BRV_0$ where K_B denotes the inverse of the flexibility matrix B. Furthermore, the static equilibrium condition balancing the left end loads and the external loads is simply

$$P_{O} = -R^{T}P = -R^{T}K_{B}\tilde{V} + R^{T}K_{B}RV_{O}$$

Separating the load vector P into two parts gives

$$P = P_E + P_F = P_E - K_F V$$

where K_F denotes the foundation stiffness matrix. Then the requirement that the foundation and the beam displacements match at x_1 , ..., x_n leads to $P_E - K_F \tilde{V} = K_B \tilde{V} - K_B R V_O$. This condition coupled with the requirement that P

should vanish gives the following system of equations which can be solved for $[\tilde{v}^T$, $v_0^T]^T$

$$\begin{bmatrix} (K_B + K_E) & -K_B R \\ -(K_B R)^T & R^T K_B R \end{bmatrix} \begin{bmatrix} \tilde{V} \\ V_O \end{bmatrix} = \begin{bmatrix} P_E \\ O \end{bmatrix}$$

The square matrix on the left side of the last equation is symmetric and the system can be solved by methods such as Cholesky decomposition commonly used in finite-element analysis. Despite the fact that a stiffness matrix approach can be used in analyzing the interaction problem of interest here, using flexibility matrices seems more natural with the governing differential equations giving deflections directly as function of loads.

Numerical Results

45. The analysis procedures developed here were implemented in two computer programs. The first program, which computes shear, moment, deflection, and interface pressure for a constant depth beam supported on an elastic halfplane, is discussed in Appendix A. The half-plane can be in plane strain or plane stress. The beam is loaded by a general combination of concentrated loads and linearly varying distributed loads. The interaction between the half-plane and the beam accounts only for vertical load components occurring at an arbitrary number of supports spaced equally between limits defined according to the data input. A system of equations determining the interaction forces is formulated and solved. The interaction forces are then replaced by distributed loads to provide a piecewise constant approximation for the foundation pressure. The interface pressure is used to compute values for the shear, moment, and deflection. Three example problems are included as typical test cases. A beam 1,200 in.* long and 120 in. deep has a modulus of 3,000 kips/in. 2 and rests on a half-plane having an elastic modulus equal to 300 kips/in.^2 and Poisson's ratio equal to 0.2. The load cases studied include:

^{*} A table of factors for converting non-SI units of measurement to SI (metric) units is presented on page 3.

- a. A 500-kip downward load is distributed uniformly over the total length. (The beam width is 1 in. measured normal to the xy-plane.
- <u>b</u>. Two concentrated downward loads, each having a magnitude of 250 kips, are applied at the beam ends.
- c. Two concentrated couples, each having moment magnitudes of 6,000 kip-in., are applied at the beam ends. The sense of the couples at the left and the right ends are clockwise and counterclockwise, respectively. The couples are simulated by using ramp loads.
- 46. Computer output for these problems appears in Appendix A. These results exhibit the type of behavior typically observed in punch problems, namely the occurrence of very large stresses near the ends of the beam. In the case of the uniformly loaded beam, the large interface pressure at the ends rapidly diminishes to a nearly uniform value near the middle of the beam. When the same external load resultant is concentrated at the ends, rather than being distributed uniformly, most of the load goes into the foundation at the ends. Because the half-plane tends to deflect downward at the midpoint more so than the beam does, a reversal in the reaction direction occurs near the beam center to maintain displacement continuity. In the third case, the two concentrated end couples have a zero statical resultant and tend to cause the beam to have compression in the top and tension in the bottom. If the beam ends were not connected to the half-plane, the ends would rise. Consequently, imposition of displacement continuity causes a tensile interface reaction at the ends and compression at the middle.
- 47. Some discussion of the approximations used to obtain distributed interface pressures should be presented. After the reactions R_1 , ..., R_n at points x_1 , ..., x_n equally spaced at a distance deare computed, then each interior reaction R_1 , $2 \le i \le n-1$ is distributed uniformly over an interval of length decentered at x_i . The left end reaction R_1 is distributed uniformly between x=0 and x=0.5d. The right end reaction is distributed similarly between $x=\ell-0.5d$ and $x=\ell$. Although this method is simple, it leads to a small moment imbalance at the ends. Since the load resultant, due to R_1 , is moved inward from the left end to a position of 0.25d, this causes a couple effect which becomes negligible when enough supports are used. A similar moment imbalance is applicable for the right end. In the case where the loading and the support positions are symmetrical about the beam center, the moment imbalance, with respect to the right end,

cancels so that the internal moment at $x = \ell$ still comes out to be zero.

48. A source listing of the first computer program used to solve the three test examples immediately follows the numerical results. The second program and numerical results are detailed and are given in Appendix B. Since both programs run interactively, the data input options are evident from the output. The results presented were obtained by use of an IBM PC/XT computer employing an Intel-8087 floating point coprocessor and a Winchester disk drive. All problems were solved in less than two min.

APPENDIX A: PROGRAM I--FOUNDATION INTERACTION BETWEEN A BEAM AND AN ELASTIC HALF-PLANE

1. Program I deals with a constant depth beam supported on an elastic half-plane, in plane strain or plane stress, and computes shear, moment, deflection, and interface pressure. Interaction forces between the half-plane and the beam account only for vertical type loads which occur at optional and equally spaced supports between certain limits defined according to the data input.

Case I--Uniform Distributed Downward Load

2. A 500-kip/ft downward pressure load is applied over the total length of the beam as shown in the following sample problem.

INPUT: Problem title (for echo check put \$ in column one)
Sample problem for WES with constant distributed load (kips,inches)

SELECT AN OPTION: 1 = Plane strain, 2 = Plane stress 1

INPUT: Young's modulus and Poisson's ratio for the half plane $300\ldots2$

INPUT: The length, the depth, and Young's modulus for the beam 1200.,120.,3000.

To define the foundation interaction points select: X-min, X-max, the number of evenly spaced support points and the width of the support in the direction of the beam axis 0.,1200.,41,1.

For the external beam loading input: The number of concentrated loads and the number of linearly varying ramp loads 0,1

and the second transform and the second of the second seco

INPUT: The starting magnitude, starting position, end magnitude, and end position for each ramp load -.416666666,0.,-.416666666,1200.

Is the foundation flexibility matrix to be printed? (Y/N) $\mbox{\scriptsize N}$

Is the foundation stiffness matrix to be printed? (Y/N) N

*** FOUNDATION FORCES AND DISPLACEMENTS ***

Reaction

Force

1.58169E+01

Displacement

in beam
[relative]

-6.83897E-14

Index Position

.00000E+00

1	.000002+00	1.301036+01	-0.0309/E-14				
2	3.00000E+01	1.25589E+01	-5.42858E-02				
3	6.00000E+01	1.14662E+01	-1.07908E-01				
4	9.00000E+01	1.10904E+01	-1.60276E-01				
5	1.20000E+02	1.10277E+01	-2.10858E-01				
6			-2.59204E-01				
7	1.80000E+02		-3.04957E-01				
8	2.10000E+02		-3.47845E-01				
9			-3.87673E-01				
10	2.70000E+02		-4.24309E-01				
11	3.00000E+02		-4.57679E-01				
12	3.30000E+02	1.21430E+01	-4.87745E-01				
13	3.60000E+02		-5.14505E-01				
14	3.90000E+02		-5.37979E-01				
15	4.20000E+02		-5.58200E-01				
16	4.50000E+02		-5.75210E-01				
17	4.80000E+02	1.25950E+01 1.26371E+01	-5.89052E-01				
18	5.10000E+02	1.263/1E+01	-5.99769E-01				
19		1.26661E+01	-6.0/396E-01				
20	5.70000E+02	1.26832E+01 1.26888E+01	-6.11961E-01				
21	6.00000E+02						
22	6.30000E+02	1.26832E+01	-6.11961E-01				
23	6.60000E+02	1.26661E+01 1.26371E+01	-6.07396E-01				
24	6.90000E+02	1.26371E+01	-5.99769E-01				
25	7.20000E+02	1.25950E+01 1.25388E+01	-5.89052E-01				
26	7.50000E+02	1.25388E+01	-5.75210E-01				
27		1.24669E+01					
28	8.10000E+02	1.23779E+01 1.22703E+01	-5.37979E-01				
29	8.40000E+02	1.22703E+01	-5.14505E-01				
30	8.70000E+02	1.21430E+01	-4.87745E-01				
31	9.00000E+02	1.19957E+01	-4.57679E-01				
32		1.18295E+01					
33	9.60000E+02	1.16478E+01 1.14575E+01	-3.87673E-01				
34	9.90000E+02	1.14575E+01	-3.47845E-01				
35	1.02000E+03	1.12720E+01	-3.04957E-01				
36	1.05000E+03	1.11146E+01	-2.59204E-01				
37	1.08000E+03	1.11146E+01 1.10277E+01	-2.10858E-01				
38	1.11000E+03	1.10904E+01					
39	1 14000E+03	1 14662E+01	-1 07908E-01				
40	1.17000E+03	1.25589E+01	-5.42858E-02				
41		1.58169E+01					
Resi	dual shear e	rror at right end:	-4.08562E-14				
Resi	dual moment e	rror at right end:	1.19691E-11				
			_ : 				
Disp	lacements are	relative to:	-1.28428E+00	at	node	#	41

To tabulate foundation pressures for a given interval: input X-min, X-max, and number of increments(input 0,0,0 to STOP) 0.,1200.,40

*** SUMMARY OF RESULTS FOR INTERFACE POINTS ***

Position	Interface Pressure	Shear	Moment	Displacement [relative]
.00000E+00	1.05446E+00	.00000E+00	.00000E+00	-6.83897E-14
3.00000E+01	4.18632E-01	9.59639E+00		
6.00000E+01		9.10894E+00	2.15477E+02	-5.42858E-02
9.00000E+01	3.82205E-01		5.00155E+02	-1.07908E-01
	3.69679E-01	7.88721E+00	7.56506E+02	-1.60276E-01
1.20000E+02	3.67590E-01	6.44625E+00	9.71743E+02	-2.10858E-01
1.50000E+02	3.70488E-01	5.01741E+00	1.14337E+03	-2.59204E-01
1.80000E+02	3.75732E-01	3.71071E+00	1.27370E+03	-3.04957E-01
2.10000E+02	3.81918E-01	2.57546E+00	1.36730E+03	-3.47845E-01
2.40000E+02	3.88259E-01	1.62811E+00	1.42964E+03	-3.87673E-01
2.70000E+02	3.94317E-01	8.66745E-01	1.46638E+03	-4.24309E-01
3.00000E+02	3.99857E-01	2.79349E-01	1.48295E+03	-4.57679E-01
3.30000E+02	4.04766E-01	-1.51311E-01	1.48432E+03	-4.87745E-01
3.60000E+02	4.09010E-01	-4.44675E-01	1.47490E+03	-5.14505E-01
3.90000E+02	4.12597E-01	-6.20570E-01	1.45852E+03	-5.37979E-01
4.20000E+02	4.15565E-01	-6.98137E-01	1.43840E+03	-5.58200E-01
4.50000E+02	4.17960E-01	-6.95266E-01	1.41723E+03	-5.75210E-01
4.80000E+02	4.19834E-01	-6.28356E-01	1.39717E+03	-5.89052E-01
5.10000E+02	4.21235E-01	-5.12314E-01	1.37990E+03	-5.99769E-01
5.40000E+02	4.22205E-01	-3.60709E-01	1.36670E+03	-6.07396E-01
5.70000E+02	4.22774E-01	-1.86027E-01	1.35843E+03	-6.11961E-01
6.00000E+02	4.22961E-01	-6.74544E-13	1.35562E+03	-6.13480E-01
6.30000E+02	4.22774E-01	1.86027E-01	1.35843E+03	-6.11961E-01
6.60000E+02	4.22205E-01	3.60709E-01	1.36670E+03	-6.07396E-01
6.90000E+02	4.21235E-01	5.12314E-01	1.37990E+03	-5.99769E-01
7.20000E+02	4.19834E-01	6.28356E-01	1.39717E+03	-5.89052E-01
7.50000E+02	4.17960E-01	6.95266E-01	1.41723E+03	-5.75210E-01
7.80000E+02	4.15565E-01	6.98137E-01	1.43840E+03	-5.58200E-01
8.10000E+02	4.12597E-01	6.20570E-01	1.45852E+03	-5.37979E-01
8.40000E+02	4.09010E-01	4.44675E-01	1.47490E+03	-5.14505E-01
8.70000E+02	4.04766E-01	1.51311E-01	1.48432E+03	-4.87745E-01
9.00000E+02	3.99857E-01	-2.79349E-01	1.48295E+03	-4.57679E-01
9.30000E+02	3.94317E-01	-8.66745E-01	1.46638E+03	-4.24309E-01
9.60000E+02	3.88259E-01	-1.62811E+00	1.42964E+03	-3.87673E-01
9.90000E+02	3.81918E-01	-2.57546E+00	1.36730E+03	-3.47845E-01
1.02000E+03	3.75732E-01	-3.71071E+00	1.27370E+03	-3.04957E-01
1.05000E+03	3.70488E-01	-5.01741E+00	1.14337E+03	-2.59204E-01
1.08000E+03	3.67590E-01	-6.44625E+00	9.71743E+02	-2.10858E-01
1.11000E+03	3.69679E-01	-7.88721E+00	7.56506E+02	-1.60276E-01
1.14000E+03	3.82205E-01	-9.10894E+00	5.00155E+02	-1.07908E-01
1.17000E+03	4.18632E-01	-9.59639E+00	2.15477E+02	-5.42858E-02
1.20000E+03	1.05446E+00	-1.51545E-14	-6.59668E-11	.00000E+00

To tabulate foundation pressures for a given interval: input X-min, X-max, and number of increments(input 0,0,0 to STOP) 0,0,0

Case II--Two Concentrated Downward Loads at Beam Ends

3. Two concentrated downward loads applied at beam ends appear in the following sample problem.

INPUT: Problem title (for echo check put \$ in column one)
Sample problem for WES with concentrated loads at ends(kips,inches)

SELECT AN OPTION: 1 = Plane strain, 2 = Plane stress

INPUT: Young's modulus and Poisson's ratio for the half plane 300.,.2

INPUT: The length, the depth, and Young's modulus for the beam 1200.,120.,3000.

To define the foundation interaction points select: X-min, X-max, the number of evenly spaced support points and the width of the support in the direction of the beam axis 0.,1200.,41,1.

For the external beam loading input: The number of concentrated loads and the number of linearly varying ramp loads 2.0

INPUT: The magnitude and position of each applied force -250.,0. -250.,1200.

Is the foundation flexibility matrix to be printed? (Y/N) \mathbf{n}

Is the foundation stiffness matrix to be printed? (Y/N) $\ensuremath{\mathtt{n}}$

*** FOUNDATION FORCES AND DISPLACEMENTS ***

Index	Position	Reaction Force	Displacement in beam [relative]
1	.00000E+00	9.47707E+01	-2.11667E+00
	3.00000E+01	5.93416E+01	-1.86392E+00
	6.00000E+01	4.08453E+01	-1.62026E+00
	9.00000E+01	2.88133E+01	-1.39187E+00
5	1.20000E+02	2.00133E+01 2.02717E+01	-1.18231E+00
	1.50000E+02	1.39668E+01	-9.93318E-01
	1.80000E+02	9,23796E+00	-8.25325E-01
8	2.10000E+02	5.67990E+00	-6.77882E-01
	2.40000E+02	3.01605E+00	-5.49948E-01
	2.70000E+02	1.04367E+00	-4.40118E-01
11	3.00000E+02	-3.92848E-01	-3.46791E-01
12	3.30000E+02	-1.41621E+00	-2.68294E-01
13	3.60000E+02	-2.12463E+00	-2.02977E-01
14	3.90000E+02	-2.59712E+00	-1.49274E-01
15	4.20000E+02	-2,89722E+00	-1.05748E-01
	4.50000E+02	-3.07562E+00	-7.11239E-02
17	4.80000E+02	-3.17223E+00	-4.43064E-02
18	5.10000E+02	-3.21772E+00	-2.43912E-02
19	5.40000E+02	-3.23477E+00	-1.06719E-02
20	5.70000E+02	-3.23894E+00	-2.64266E-03
21	6.00000E+02	-3.23934E+00	.00000E+00
22	6.30000E+02	-3.23894E+00	-2.64266E-03
23	6.60000E+02	-3.23477E+00	-1.06719E-02
24	6.90000E+02	-3.21772E+00	-2.43912E-02
25	7.20000E+02	-3.17223E+00	-4.43064E-02
26	7.50000E+02	-3.07562E+00	-7.11239E-02
27	7.80000E+02	-2.89722E+00	-1.05748E-01
28	8.10000E+02	-2.59712E+00	-1.49274E-01
29	8.40000E+02	-2.12463E+00	-2.02977E-01
30	8.70000E+02	-1.41621E+00	-2.68294E-01
31	9.00000E+02	-3.92848E-01	-3.46791E-01
32	9.30000E+02	1.04367E+00	-4.40118E-01
33	9.60000E+02	3.01605E+00	-5.49948E-01
34	9.90000E+02	5.67990E+00	-6.77882E-01
3 5	1.02000E+03	9.23796E+00	-8.25325E-01
36	1.05000E+03	1.39668E+01	-9.93318E-01
37	1.08000E+03	2.02717E+01	-1.18231E+00
38	1.11000E+03	2.88133E+01	-1.39187E+00
39	1.14000E+03	4.08453E+01	-1.62026E+00
40	1.17000E+03	5.93416E+01	-1.86392E+00
41	1.20000E+03	9.47707E+01	-2.11667E+00

Residual shear error at right end: 9.94760E-14
Residual moment error at right end: 3.99893E-11

Displacements are relative to: -6.93861E-01 at node # 21

To tabulate foundation pressures for a given interval: input X-min, X-max, and number of increments(input 0,0,0 to STOP) 0.,1200.,40

*** SUMMARY OF RESULTS FOR INTERFACE POINTS ***

Pressure	Position	Interface	Shear	Moment	Displacement
3.00000E+01 1.97805E+00 -1.25559E+02 -5.14513E+03 -1.86392E+00 6.00000E+01 9.60443E-01 -4.06358E+01 -9.78751E+03 -1.62026E+00 1.20000E+02 6.75723E-01 -1.60933E+01 -1.06064E+04 -1.18231E+00 1.50000E+02 4.65560E-01 1.02599E+00 -1.08088E+04 -9.93318E-01 1.80000E+02 3.07932E-01 1.26284E+01 -1.05862E+04 -8.25325E-01 2.10000E+02 1.89330E-01 2.00873E+01 -1.08082E+04 -8.25325E-01 2.40000E+02 1.00535E-01 2.44535E+01 -9.40433E+03 -5.49948E-01 2.70000E+02 3.47889E-02 2.664651E+01 -8.63342E+03 -4.40118E-01 3.00000E+02 -1.30949E-02 2.67905E+01 -7.82920E+03 -3.46791E-01 3.30000E+02 -7.08209E-02 2.41156E+01 -6.28254E+03 -2.68294E-01 4.20000E+02 -8.65708E-02 2.17547E+01 -5.59271E+03 -1.49274E-01 4.20000E+02 -1.02521E-01 1.60211E+01 -4.45405E+03 -1.05748E-01 5.10000E+02 -1.07257E-01 9.70223E+00 -3.68075E+03 -2.64266E-03 6.0000E+02 -1.07965E-01 3.23914E+00 -3.29227E+03 -2.64266E-03 6.0000E+02 -1.07965E-01 -3.23914E+00 -3.29227E+03 -2.64266E-03 6.0000E+02 -1.07976E-01 -1.2171E-12 -3.24368E+03 -1.06719E-02 7.50000E+02 -1.07571E-01 -9.70223E+00 -3.68075E+03 -7.11239E-02 7.50000E+02 -1.07571E-01 -9.70223E+00 -3.68075E+03 -1.06719E-02 7.50000E+02 -1.07571E-01 -9.70223E+00 -3.68075E+03 -2.64266E-03 6.0000E+02 -1.07965E-01 -3.23914E+00 -3.29227E+03 -2.64266E-03 6.0000E+02 -1.07571E-01 -6.47599E+00 -3.43801E+03 -1.06719E-02 7.50000E+02 -1.07571E-01 -6.47599E+00 -3.43801E+03 -1.06719E-02 7.50000E+02 -1.07571E-01 -6.47599E+00 -3.68075E+03 -7.11239E-02 7.80000E+02 -1.07571E-01 -6.47599E+00 -3.43801E+03 -1.06719E-02 7.5000E+02 -1.07571E-01 -6.47599E+00 -3.43801E+03 -1.06719E-02 7.5000E+02 -1.07571E-01 -6.475				22222	
6.00000E+01 1.36151E+00 -7.54650E+01 -8.09112E+03 -1.62026E+00 9.0000E+01 9.60443E-01 -4.06358E+01 -9.78751E+03 -1.39187E+00 1.20000E+02 6.75723E-01 -1.60933E+01 -1.06064E+04 -1.18231E+00 1.50000E+02 4.65560E-01 1.02599E+00 -1.08088E+04 -9.93318E-01 2.10000E+02 1.89330E-01 2.00873E+01 -1.05862E+04 -8.25325E-01 2.40000E+02 1.00535E-01 2.00873E+01 -1.00822E+04 -6.77882E-01 2.70000E+02 3.47889E-02 2.64651E+01 -8.63342E+03 -5.49948E-01 3.0000E+02 -1.30949E-02 2.67905E+01 -7.82920E+03 -3.46791E-01 3.0000E+02 -1.30949E-02 2.58860E+01 -7.03522E+03 -2.68294E-01 3.6000E+02 -7.08209E-02 2.41156E+01 -6.28254E+03 -2.02977E-01 4.2000E+02 -9.65741E-02 1.90076E+01 -4.98015E+03 -1.05748E-01 4.9000E+02 -1.07257E-01 1.28972E+01 -4.04505E+03 -1.05748E-01 4.8000E+02 -1.07257E-01 1.28972E+01 -4.01991E+03 -4.43064E-02 5.70000E+02 -1.07826E-01 6.47599E+00 -3.48301E+03 -1.06719E-02 5.70000E+02 -1.07965E-01 3.23914E+00 -3.29227E+03 -2.64266E-03 6.0000E+02 -1.07757E-01 -9.70223E+00 -3.68075E+03 -2.43912E-02 6.9000E+02 -1.077826E-01 -6.47599E+00 -3.43801E+03 -1.06719E-02 6.90000E+02 -1.077826E-01 -3.23914E+00 -3.29227E+03 -2.64266E-03 6.0000E+02 -1.077826E-01 -3.23914E+00 -3.29227E+03 -2.64266E-03 6.0000E+02 -1.07757E-01 -9.70223E+00 -3.68075E+03 -2.43912E-02 7.2000E+02 -1.07757E-01 -9.70223E+00 -3.68075E+03 -2.43912E-02 7.2000E+02 -1.07826E-01 -6.47599E+00 -3.43801E+03 -1.06719E-02 7.50000E+02 -1.07257E-01 -9.70223E+00 -3.68075E+03 -2.43912E-02 7.20000E+02 -1.07257E-01 -9.70223E+00 -3.68075E+03 -2.43912E-02 7.20000E+02 -1.075741E-01 -1.60211E+01 -4.45405E+03 -1.06719E-02 7.50000E+02 -1.075741E-01 -1.60211E+01 -4.45405E+03 -1.06719E-02 7.5000E+02 -1.075741E-01 -1.60211E+01 -4.45405E+03 -1.06719E-02 7.5000E+02 -1.075741E-01 -1.60211E+01 -4.45405E+03 -1.06719E-02 7.5000E+02 -1.075741E-01 -1.60211E+01 -4.55405E+03 -1.06719E-02 7.5000E+02 -1.075741E-01 -1.60211E+01 -4.55405E+03 -1.06719E-02 7.5000E+02 -1.075741E-01 -1.60211E+01 -4.55405E+03 -1.06719E-02 7.5000E+02 -1.05741E-01 -1.60211E+01 -4.55405E+03 -1.06719E-01 7.49070E-02 -2.43150E-01 -					
9.00000E+01 9.60443E-01 -4.06358E+01 -9.78751E+03 -1.39187E+00 1.20000E+02 6.75723E-01 -1.60933E+01 -1.06064E+04 -1.18231E+00 1.50000E+02 4.65560E-01 1.02599E+00 -1.08088E+04 -9.93318E-01 1.80000E+02 3.07932E-01 1.26284E+01 -1.05862E+04 -8.25325E-01 2.10000E+02 1.89330E-01 2.00873E+01 -1.00822E+04 -6.77882E-01 2.40000E+02 1.00535E-01 2.44353E+01 -9.40433E+03 -5.49948E-01 2.70000E+02 3.47889E-02 2.66651E+01 -8.63342E+03 -3.46791E-01 3.0000E+02 -1.30949E-02 2.58860E+01 -7.03522E+03 -3.46791E-01 3.0000E+02 -7.08209E-02 2.41156E+01 -6.28254E+03 -2.02977E-01 3.90000E+02 -8.65708E-02 2.17547E+01 -5.59271E+03 -1.05748E-01 4.50000E+02 -1.05741E-01 1.28972E+01 -4.01991E+03 -4.43064E-02 5.10000E+02 -1.07257E-01 9.70223E+00 -3.68075E+03 -2.43912E-02 5.40000E+02 -1.07965E-01 3.23914E+00 -3.29227E+03 -2.64266E-03 6.0000E+02 -1.07965E-01 -3.23914E+00 -3.29227E+03 -2.64266E-03 6.0000E+02 -1.07965E-01 -1.12171E-12 -3.24368E+03 -1.06719E-02 7.5000E+02 -1.07257E-01 -9.70223E+00 -3.68075E+03 -2.43912E-02 7.5000E+02 -1.07257E-01 -9.70223E+00 -3.68075E+03 -2.43912E-02 7.5000E+02 -1.07257E-01 -9.70223E+00 -3.43801E+03 -1.06719E-02 6.9000E+02 -1.07257E-01 -9.70223E+00 -3.43801E+03 -1.06719E-02 7.5000E+02 -1.07257E-01 -9.70223E+00 -3.43801E+03 -1.06719E-02 7.5000E+02 -1.07257E-01 -9.70223E+00 -3.68075E+03 -2.64266E-03 6.6000E+02 -1.07826E-01 -6.47599E+00 -3.43801E+03 -1.06719E-02 7.5000E+02 -1.0736E-01 -1.28972E+01 -4.01991E+03 -4.43064E-02 7.5000E+02 -1.0736E-01 -7.08209E-02 -2.41156E+01 -6.28254E+03 -2.02977E-01 8.40000E+02 -9.65741E-02 -1.90076E+01 -4.98015E+03 -1.06719E-02 9.5000E+02 -7.08209E-02 -2.41156E+01 -6.28254E+03 -2.02977E-01 9.0000E+02 -1.0935E-01 -2.44353E+01 -9.40433E+03 -4.40118E-01 9.0000E+02 -1.0935E-01 -2.					
1.20000E+02	6.00000E+01				
1.50000E+02					
1.80000E+02	1,20000E+02			· · ·	
2.10000E+02	1,50000E+02				
2.40000E+02	1.80000E+02	3.07932E-01			
2.70000E+02 3.47889E-02 2.64651E+01 -8.63342E+03 -4.40118E-01 3.00000E+02 -1.30949E-02 2.67905E+01 -7.82920E+03 -3.46791E-01 3.30000E+02 -4.72070E-02 2.58860E+01 -7.03522E+03 -2.68294E-01 3.60000E+02 -7.08209E-02 2.41156E+01 -6.28254E+03 -2.02977E-01 3.90000E+02 -8.65708E-02 2.17547E+01 -5.59271E+03 -1.49274E-01 4.20000E+02 -9.65741E-02 1.90076E+01 -4.98015E+03 -1.05748E-01 4.50000E+02 -1.02521E-01 1.60211E+01 -4.45405E+03 -7.11239E-02 4.80000E+02 -1.075741E-01 1.28972E+01 -4.01991E+03 -4.43064E-02 5.10000E+02 -1.07826E-01 6.47599E+00 -3.43801E+03 -1.06719E-02 5.70000E+02 -1.07965E-01 3.23914E+00 -3.29227E+03 -2.64266E-03 6.0000E+02 -1.07978E-01 -1.12171E-12 -3.24368E+03 .00000E+02 6.30000E+02 -1.07965E-01 -3.23914E+00 -3.29227E+03 -2.64266E-03 6.60000E+02 -1.07965E-01 -3.23914E+00 -3.29227E+03 -2.64266E-03 6.60000E+02 -1.07965E-01 -3.23914E+00 -3.29227E+03 -2.64266E-03 6.60000E+02 -1.07965E-01 -3.23914E+00 -3.29227E+03 -2.64266E-03 6.50000E+02 -1.07965E-01 -3.23914E+00 -3.29227E+03 -2.64266E-03 6.80000E+02 -1.07965E-01 -3.23914E+00 -3.99227E+03 -2.64266E-03 6.80000E+02 -1.07965E-01 -3.23914E+00 -3.99227E+03 -2.64266E-03 6.80000E+02 -1.07965E-01 -3.23914E+00 -3.586075E+03 -2.43912E-02 7.50000E+02 -1.07574E-01 -9.70223E+00 -3.68075E+03 -2.43912E-02 7.50000E+02 -1.05741E-01 -1.28972E+01 -4.01991E+03 -4.43064E-02 7.50000E+02 -1.05741E-01 -1.28972E+01 -4.01991E+03 -4.43064E-02 7.50000E+02 -1.05741E-01 -1.28972E+01 -4.01991E+03 -4.43064E-02 7.50000E+02 -1.05741E-01 -1.06211E+01 -4.45405E+03 -1.05748E-01 8.10000E+02 -8.65708E-02 -2.17547E+01 -5.59271E+03 -1.05748E-01 8.10000E+02 -8.65708E-02 -2.17547E+01 -5.59271E+03 -1.05748E-01 9.30000E+02 -4.72070E-02 -2.58860E+01 -7.03522E+03 -2.68294E-01 9.30000E+02 -1.00535E-01 -2.44353E+01 -8.63342E+03 -4.40118E-01 9.30000E+02 1.00535E-01 -2.44353E+01 -9.40433E+03 -5.49948E-01 9.30000E+02 1.00535E-01 -2.44353E+01 -1.0082E+04 -6.77882E-01 1.02000E+03 3.07932E-01 -1.26284E+01 -1.05862E+04 -8.25325E-01	2,10000E+02	1.89330E-01			
3.00000E+02 -1.30949E-02 2.67905E+01 -7.82920E+03 -3.46791E-01 3.30000E+02 -4.72070E-02 2.58860E+01 -7.03522E+03 -2.68294E-01 3.60000E+02 -7.08209E-02 2.41156E+01 -6.28254E+03 -2.02977E-01 3.90000E+02 -8.65708E-02 2.17547E+01 -5.59271E+03 -1.49274E-01 4.20000E+02 -9.65741E-02 1.90076E+01 -4.98015E+03 -1.05748E-01 4.50000E+02 -1.02521E-01 1.60211E+01 -4.45405E+03 -7.11239E-02 4.80000E+02 -1.075741E-01 1.28972E+01 -4.01991E+03 -4.43064E-02 5.10000E+02 -1.07257E-01 9.70223E+00 -3.68075E+03 -2.43912E-02 5.40000E+02 -1.07826E-01 6.47599E+00 -3.43801E+03 -1.06719E-02 5.70000E+02 -1.07965E-01 3.23914E+00 -3.29227E+03 -2.64266E-03 6.0000E+02 -1.07965E-01 3.23914E+00 -3.29227E+03 -2.64266E-03 6.0000E+02 -1.07965E-01 -1.12171E-12 -3.24368E+03 .00000E+00 6.30000E+02 -1.07826E-01 -3.23914E+00 -3.29227E+03 -2.64266E-03 6.9000E+02 -1.07965E-01 -9.70223E+00 -3.43801E+03 -1.06719E-02 6.9000E+02 -1.07576E-01 -9.70223E+00 -3.43801E+03 -1.06719E-02 7.20000E+02 -1.075741E-01 -9.70223E+00 -3.68075E+03 -2.43912E-02 7.50000E+02 -1.05741E-01 -1.28972E+01 -4.01991E+03 -4.43064E-02 7.50000E+02 -1.05741E-01 -1.28972E+01 -4.01991E+03 -4.43064E-02 7.80000E+02 -9.65741E-02 -1.90076E+01 -4.98015E+03 -7.11239E-02 8.40000E+02 -8.65708E-02 -2.17547E+01 -5.59271E+03 -1.49274E-01 8.40000E+02 -8.65708E-02 -2.17547E+01 -5.59271E+03 -1.49274E-01 8.40000E+02 -4.72070E-02 -2.58860E+01 -7.03522E+03 -2.68294E-01 9.0000E+02 -1.30949E-02 -2.67905E+01 -7.03522E+03 -2.68294E-01 9.0000E+02 1.00535E-01 -2.44353E+01 -9.40433E+03 -4.40118E-01 9.60000E+02 1.00535E-01 -2.44353E+01 -9.40433E+03 -5.49948E-01 9.90000E+02 1.89330E-01 -2.00873E+01 -1.0082E+04 -6.77882E-01 1.02000E+03 3.07932E-01 -1.26284E+01 -1.05862E+04 -8.25325E-01 1.02000E+03 3.07932E-01 -1.26284E+01 -1.05862E+04 -8.25325E-01	2.40000E+02	1.00535E-01	2.44353E+01		
3.30000E+02	2.70000E+02	3.47889E-02	2.64651E+01	-8.63342E+03	-4.40118E-01
3.60000E+02	3.00000E+02	-1.30949E-02	2.67905E+01	-7.82920E+03	-3.46791E-01
3.90000E+02	3.30000E+02	-4.72070E-02	2,58860E+01	-7.03522E+03	-2.68294E-01
3.90000E+02	3.60000E+02	-7.08209E-02	2.41156E+01	-6.28254E+03	-2.02977E-01
4.50000E+02 -1.02521E-01 1.60211E+01 -4.45405E+03 -7.11239E-02 4.80000E+02 -1.05741E-01 1.28972E+01 -4.01991E+03 -4.43064E-02 5.10000E+02 -1.07257E-01 9.70223E+00 -3.68075E+03 -2.43912E-02 5.40000E+02 -1.07826E-01 6.47599E+00 -3.43801E+03 -1.06719E-02 5.70000E+02 -1.07965E-01 3.23914E+00 -3.29227E+03 -2.64266E-03 6.00000E+02 -1.07978E-01 -1.12171E-12 -3.24368E+03 .00000E+00 6.30000E+02 -1.07965E-01 -3.23914E+00 -3.29227E+03 -2.64266E-03 6.60000E+02 -1.07826E-01 -6.47599E+00 -3.43801E+03 -1.06719E-02 6.90000E+02 -1.07257E-01 -9.70223E+00 -3.68075E+03 -2.43912E-02 7.20000E+02 -1.05741E-01 -1.28972E+01 -4.01991E+03 -4.43064E-02 7.50000E+02 -1.02521E-01 -1.60211E+01 -4.45405E+03 -7.11239E-02 7.80000E+02 -9.65741E-02 -1.90076E+01 -4.98015E+03 -1.05748E-01 8.40000E+02 -8.65708E-02 -2.17547E+01 -5.59271E+03 -1.49274E-01 8.40000E+02 -7.08209E-02 -2.41156E+01 -6.28254E+03 -2.02977E-01 8.70000E+02 -4.72070E-02 -2.58860E+01 -7.03522E+03 -2.68294E-01 9.00000E+02 3.47889E-02 -2.64651E+01 -8.63342E+03 -4.40118E-01 9.30000E+02 1.00535E-01 -2.44353E+01 -9.40433E+03 -5.49948E-01 9.90000E+02 1.89330E-01 -2.00873E+01 -1.00822E+04 -6.77882E-01 1.02000E+03 3.07932E-01 -1.26284E+01 -1.05862E+04 -8.25325E-01		-8.65708E-02	2.17547E+01	-5.59271E+03	-1.49274E-01
4.80000E+02 -1.05741E-01	4.20000E+02	-9.65741E-02	1.90076E+01	-4.98015E+03	-1.05748E-01
4.80000E+02 -1.05741E-01 1.28972E+01 -4.01991E+03 -4.43064E-02 -1.07000E+02 -1.07257E-01 9.70223E+00 -3.68075E+03 -2.43912E-02 -5.40000E+02 -1.07826E-01 6.47599E+00 -3.43801E+03 -1.06719E-02 -7.0000E+02 -1.07978E-01 -1.12171E-12 -3.24368E+03 .00000E+02 -1.07978E-01 -1.12171E-12 -3.24368E+03 .00000E+00 -1.07965E-01 -3.23914E+00 -3.29227E+03 -2.64266E-03 6.60000E+02 -1.07826E-01 -6.47599E+00 -3.43801E+03 -1.06719E-02 -7.0000E+02 -1.07257E-01 -9.70223E+00 -3.68075E+03 -2.43912E-02 -7.20000E+02 -1.05741E-01 -1.28972E+01 -4.01991E+03 -4.43064E-02 -7.50000E+02 -1.02521E-01 -1.60211E+01 -4.45405E+03 -7.11239E-02 -7.80000E+02 -9.65741E-02 -1.90076E+01 -4.98015E+03 -1.05748E-01 -1.05748E-01 -1.0000E+02 -7.08209E-02 -2.41156E+01 -5.59271E+03 -1.49274E-01 -7.08209E-02 -2.58860E+01 -7.03522E+03 -2.68294E-01 -7.035000E+02 -1.30949E-02 -2.58860E+01 -7.03522E+03 -2.68294E-01 -7.82920E+03 -3.46791E-01 -7.82920E+03 -3.46791E-01 -7.82920E+03 -3.46791E-01 -7.990000E+02 1.00535E-01 -2.44353E+01 -9.40433E+03 -5.49948E-01 -7.7882E-01 -1.02000E+03 3.07932E-01 -1.26284E+01 -1.05862E+04 -8.25325E-01 -0.2325E-01 -1.26284E+01 -1.05862E+04 -8.25325E-01 -8.2325E-01 -1.26284E+01 -1.05862E+04 -8.25325E-01 -8.2325E-01 -1.26284E+01 -1.05862E+04 -8.25325E-01	4.50000E+02	-1.02521E-01	1.60211E+01	-4.45405E+03	-7.11239E-02
5.10000E+02 -1.07257E-01 9.70223E+00 -3.68075E+03 -2.43912E-02 5.40000E+02 -1.07826E-01 6.47599E+00 -3.43801E+03 -1.06719E-02 5.70000E+02 -1.07965E-01 3.23914E+00 -3.29227E+03 -2.64266E-03 6.00000E+02 -1.07978E-01 -1.12171E-12 -3.24368E+03 .00000E+00 6.30000E+02 -1.07965E-01 -3.23914E+00 -3.29227E+03 -2.64266E-03 6.60000E+02 -1.07826E-01 -6.47599E+00 -3.43801E+03 -1.06719E-02 6.90000E+02 -1.07257E-01 -9.70223E+00 -3.68075E+03 -2.43912E-02 7.20000E+02 -1.05741E-01 -1.28972E+01 -4.01991E+03 -4.43064E-02 7.50000E+02 -1.02521E-01 -1.60211E+01 -4.45405E+03 -7.11239E-02 7.80000E+02 -9.65741E-02 -1.90076E+01 -4.98015E+03 -1.05748E-01 8.10000E+02 -8.65708E-02 -2.17547E+01 -5.59271E+03 -1.49274E-01 8.40000E+02 -7.08209E-02 -2.41156E+01 -6.28254E+03 -2.02977E-01 8.70000E+02 -1.30949E-02 -2.67905E+01 -7.03522E+03 -2.68294E-01 9.30000E+02 3.47889E-02 -2.64651E+01 -8.63342E+03 -4.40118E-01 9.60000E+02 1.89330E-01 -2.00873E+01 -1.05862E+04 -6.77882E-01 1.02000E+03 3.07932E-01 -1.26284E+01 -1.05862E+04 -8.25325E-01		-1.05741E-01	1.28972E+01	-4.01991E+03	-4.43064E-02
5.40000E+02 -1.07826E-01 6.47599E+00 -3.43801E+03 -1.06719E-02 5.70000E+02 -1.07965E-01 3.23914E+00 -3.29227E+03 -2.64266E-03 6.0000E+02 -1.07978E-01 -1.12171E-12 -3.24368E+03 .00000E+00 6.30000E+02 -1.07826E-01 -3.23914E+00 -3.29227E+03 -2.64266E-03 6.60000E+02 -1.07826E-01 -6.47599E+00 -3.43801E+03 -1.06719E-02 6.90000E+02 -1.07257E-01 -9.70223E+00 -3.68075E+03 -2.43912E-02 7.20000E+02 -1.05741E-01 -1.28972E+01 -4.01991E+03 -4.43064E-02 7.50000E+02 -1.02521E-01 -1.60211E+01 -4.45405E+03 -7.11239E-02 7.80000E+02 -9.65741E-02 -1.90076E+01 -4.98015E+03 -1.05748E-01 8.10000E+02 -8.65708E-02 -2.17547E+01 -5.59271E+03 -1.49274E-01 8.40000E+02 -7.08209E-02 -2.41156E+01 -6.28254E+03 -2.02977E-01 8.70000E+02 -4.72070E-02 -2.58860E+01 -7.03522E+03 -2.68294E-01 9.30000E+02 -1.30949E-02 -2.67905E+01 -7.82920E+03 -3.46791E-01 9.30000E+02 1.00535E-01 -2.44353E+01 -9.40433E+03 -5.49948E-01 9.90000E+02 1.89330E-01 -2.00873E+01 -1.00822E+04 -6.77882E-01 1.02000E+03 3.07932E-01 -1.26284E+01 -1.0082E+04 -8.25325E-01		-1.07257E-01	9.70223E+00	-3,68075E+03	-2.43912E-02
5.70000E+02 -1.07965E-01		-1.07826E-01	6.47599E+00	-3.43801E+03	-1.06719E-02
6.0000E+02			3.23914E+00	-3.29227E+03	-2.64266E-03
6.30000E+02 -1.07965E-01 -3.23914E+00 -3.29227E+03 -2.64266E-03 6.60000E+02 -1.07826E-01 -6.47599E+00 -3.43801E+03 -1.06719E-02 6.90000E+02 -1.07257E-01 -9.70223E+00 -3.68075E+03 -2.43912E-02 7.20000E+02 -1.05741E-01 -1.28972E+01 -4.01991E+03 -4.43064E-02 7.50000E+02 -1.02521E-01 -1.60211E+01 -4.45405E+03 -7.11239E-02 7.80000E+02 -9.65741E-02 -1.90076E+01 -4.98015E+03 -1.05748E-01 8.10000E+02 -8.65708E-02 -2.17547E+01 -5.59271E+03 -1.49274E-01 8.40000E+02 -7.08209E-02 -2.41156E+01 -6.28254E+03 -2.02977E-01 9.00000E+02 -1.30949E-02 -2.58860E+01 -7.03522E+03 -2.68294E-01 9.30000E+02 -1.30949E-02 -2.64651E+01 -8.63342E+03 -3.46791E-01 9.30000E+02 1.00535E-01 -2.44353E+01 -9.40433E+03 -5.49948E-01 9.90000E+02 1.89330E-01 -2.00873E+01 -1.00822E+04 -6.77882E-01 1.02000E+03 3.07932E-01 -1.26284E+01 -1.05862E+04 -8.25325E-01 1.02000E+03 3.07932E-01 -1.26284E+01 -1.05862E+04 -8.25325E-01		_ ·	-1.12171E-12	-3.24368E+03	.00000E+00
6.60000E+02 -1.07826E-01 -6.47599E+00 -3.43801E+03 -1.06719E-02 6.90000E+02 -1.07257E-01 -9.70223E+00 -3.68075E+03 -2.43912E-02 7.20000E+02 -1.05741E-01 -1.28972E+01 -4.01991E+03 -4.43064E-02 7.50000E+02 -1.02521E-01 -1.60211E+01 -4.45405E+03 -7.11239E-02 7.80000E+02 -9.65741E-02 -1.90076E+01 -4.98015E+03 -1.05748E-01 8.10000E+02 -8.65708E-02 -2.17547E+01 -5.59271E+03 -1.49274E-01 8.40000E+02 -7.08209E-02 -2.41156E+01 -6.28254E+03 -2.02977E-01 8.70000E+02 -4.72070E-02 -2.58860E+01 -7.03522E+03 -2.68294E-01 9.00000E+02 -1.30949E-02 -2.67905E+01 -7.82920E+03 -3.46791E-01 9.30000E+02 3.47889E-02 -2.64651E+01 -8.63342E+03 -4.40118E-01 9.60000E+02 1.00535E-01 -2.44353E+01 -9.40433E+03 -5.49948E-01 9.90000E+02 1.89330E-01 -2.00873E+01 -1.00822E+04 -6.77882E-01 1.02000E+03 3.07932E-01 -1.26284E+01 -1.05862E+04 -8.25325E-01			-3.23914E+00	-3,29227E+03	-2.64266E-03
6.90000E+02 -1.07257E-01 -9.70223E+00 -3.68075E+03 -2.43912E-02 -2.20000E+02 -1.05741E-01 -1.28972E+01 -4.01991E+03 -4.43064E-02 -4.50000E+02 -1.02521E-01 -1.60211E+01 -4.45405E+03 -7.11239E-02 -7.80000E+02 -9.65741E-02 -1.90076E+01 -4.98015E+03 -1.05748E-01 -1.05748E-01 -4.98015E+03 -1.05748E-01 -4.98000E+02 -8.65708E-02 -2.17547E+01 -5.59271E+03 -1.49274E-01 -5.59271E+03 -1.49274E-01 -2.02977E-01 -2.58860E+01 -7.03522E+03 -2.02977E-01 -2.58860E+01 -7.03522E+03 -2.68294E-01 -9.0000E+02 -1.30949E-02 -2.67905E+01 -7.82920E+03 -3.46791E-01 -3.9000E+02 -1.30949E-02 -2.64651E+01 -8.63342E+03 -4.40118E-01 -9.90000E+02 1.89330E-01 -2.44353E+01 -9.40433E+03 -5.49948E-01 -5.49948E-01 -1.02000E+03 3.07932E-01 -1.26284E+01 -1.05862E+04 -8.25325E-01 -8.25325E-01			-6.47599E+00	-3.43801E+03	-1.06719E-02
7.20000E+02 -1.05741E-01 -1.28972E+01 -4.01991E+03 -4.43064E-02 7.50000E+02 -1.02521E-01 -1.60211E+01 -4.45405E+03 -7.11239E-02 7.80000E+02 -9.65741E-02 -1.90076E+01 -4.98015E+03 -1.05748E-01 8.10000E+02 -8.65708E-02 -2.17547E+01 -5.59271E+03 -1.49274E-01 8.40000E+02 -7.08209E-02 -2.41156E+01 -6.28254E+03 -2.02977E-01 8.70000E+02 -4.72070E-02 -2.58860E+01 -7.03522E+03 -2.68294E-01 9.00000E+02 -1.30949E-02 -2.67905E+01 -7.82920E+03 -3.46791E-01 9.30000E+02 3.47889E-02 -2.64651E+01 -8.63342E+03 -4.40118E-01 9.60000E+02 1.00535E-01 -2.44353E+01 -9.40433E+03 -5.49948E-01 9.90000E+02 1.89330E-01 -2.00873E+01 -1.00822E+04 -6.77882E-01 1.02000E+03 3.07932E-01 -1.26284E+01 -1.05862E+04 -8.25325E-01			-9.70223E+00	-3.68075E+03	-2.43912E-02
7.50000E+02 -1.02521E-01 -1.60211E+01 -4.45405E+03 -7.11239E-02 7.80000E+02 -9.65741E-02 -1.90076E+01 -4.98015E+03 -1.05748E-01 8.10000E+02 -8.65708E-02 -2.17547E+01 -5.59271E+03 -1.49274E-01 8.40000E+02 -7.08209E-02 -2.41156E+01 -6.28254E+03 -2.02977E-01 8.70000E+02 -4.72070E-02 -2.58860E+01 -7.03522E+03 -2.68294E-01 9.00000E+02 -1.30949E-02 -2.67905E+01 -7.82920E+03 -3.46791E-01 9.30000E+02 3.47889E-02 -2.64651E+01 -8.63342E+03 -4.40118E-01 9.60000E+02 1.00535E-01 -2.44353E+01 -9.40433E+03 -5.49948E-01 9.90000E+02 1.89330E-01 -2.00873E+01 -1.00822E+04 -6.77882E-01 1.02000E+03 3.07932E-01 -1.26284E+01 -1.05862E+04 -8.25325E-01				-4,01991E+03	-4.43064E-02
7.80000E+02 -9.65741E-02 -1.90076E+01 -4.98015E+03 -1.05748E-01 8.10000E+02 -8.65708E-02 -2.17547E+01 -5.59271E+03 -1.49274E-01 8.40000E+02 -7.08209E-02 -2.41156E+01 -6.28254E+03 -2.02977E-01 8.70000E+02 -4.72070E-02 -2.58860E+01 -7.03522E+03 -2.68294E-01 9.00000E+02 -1.30949E-02 -2.67905E+01 -7.82920E+03 -3.46791E-01 9.30000E+02 3.47889E-02 -2.64651E+01 -8.63342E+03 -4.40118E-01 9.60000E+02 1.00535E-01 -2.44353E+01 -9.40433E+03 -5.49948E-01 9.90000E+02 1.89330E-01 -2.00873E+01 -1.00822E+04 -6.77882E-01 1.02000E+03 3.07932E-01 -1.26284E+01 -1.05862E+04 -8.25325E-01		•	-1.60211E+01	-4.45405E+03	-7.11239E-02
8.10000E+02 -8.65708E-02 -2.17547E+01 -5.59271E+03 -1.49274E-01 8.40000E+02 -7.08209E-02 -2.41156E+01 -6.28254E+03 -2.02977E-01 8.70000E+02 -4.72070E-02 -2.58860E+01 -7.03522E+03 -2.68294E-01 9.00000E+02 -1.30949E-02 -2.67905E+01 -7.82920E+03 -3.46791E-01 9.30000E+02 3.47889E-02 -2.64651E+01 -8.63342E+03 -4.40118E-01 9.60000E+02 1.00535E-01 -2.44353E+01 -9.40433E+03 -5.49948E-01 9.90000E+02 1.89330E-01 -2.00873E+01 -1.00822E+04 -6.77882E-01 1.02000E+03 3.07932E-01 -1.26284E+01 -1.05862E+04 -8.25325E-01			-1.90076E+01	-4.98015E+03	-1.05748E-01
8.40000E+02			-2.17547E+01	-5.59271E+03	-1.49274E-01
8.70000E+02 -4.72070E-02 -2.58860E+01 -7.03522E+03 -2.68294E-01 9.00000E+02 -1.30949E-02 -2.67905E+01 -7.82920E+03 -3.46791E-01 9.30000E+02 3.47889E-02 -2.64651E+01 -8.63342E+03 -4.40118E-01 9.60000E+02 1.00535E-01 -2.44353E+01 -9.40433E+03 -5.49948E-01 9.90000E+02 1.89330E-01 -2.00873E+01 -1.00822E+04 -6.77882E-01 1.02000E+03 3.07932E-01 -1.26284E+01 -1.05862E+04 -8.25325E-01			-2.41156E+01	-6.28254E+03	-2.02977E-01
9.00000E+02 -1.30949E-02 -2.67905E+01 -7.82920E+03 -3.46791E-01 9.30000E+02 3.47889E-02 -2.64651E+01 -8.63342E+03 -4.40118E-01 9.60000E+02 1.00535E-01 -2.44353E+01 -9.40433E+03 -5.49948E-01 9.90000E+02 1.89330E-01 -2.00873E+01 -1.00822E+04 -6.77882E-01 1.02000E+03 3.07932E-01 -1.26284E+01 -1.05862E+04 -8.25325E-01			-2.58860E+01	-7.03522E+03	-2.68294E-01
9.30000E+02 3.47889E-02 -2.64651E+01 -8.63342E+03 -4.40118E-01 9.60000E+02 1.00535E-01 -2.44353E+01 -9.40433E+03 -5.49948E-01 9.90000E+02 1.89330E-01 -2.00873E+01 -1.00822E+04 -6.77882E-01 1.02000E+03 3.07932E-01 -1.26284E+01 -1.05862E+04 -8.25325E-01				-7.82920E+03	-3.46791E-01
9.60000E+02 1.00535E-01 -2.44353E+01 -9.40433E+03 -5.49948E-01 9.90000E+02 1.89330E-01 -2.00873E+01 -1.00822E+04 -6.77882E-01 1.02000E+03 3.07932E-01 -1.26284E+01 -1.05862E+04 -8.25325E-01			-2.64651E+01	-8.63342E+03	-4.40118E-01
9.90000E+02 1.89330E-01 -2.00873E+01 -1.00822E+04 -6.77882E-01 1.02000E+03 3.07932E-01 -1.26284E+01 -1.05862E+04 -8.25325E-01			-2.44353E+01	-9.40433E+03	-5.49948E-01
1.02000E+03 3.07932E-01 -1.26284E+01 -1.05862E+04 -8.25325E-01			-2.00873E+01	-1.00822E+04	-6.77882E-01
1 00000 00 1 00000 00 0 0 0000 00 0 0 0 0 0 0 0 0 0			-1.26284E+01	-1.05862E+04	-8.25325E-01
1.U NOOUETUJ 4.UJJUULTUL -1.VUJJIJUU -1.UVUJUTUL -1.UVUJUTULU -1.UVUJUTU -1.U	1.05000E+03	4.65560E-01	-1.02599E+00	-1.08088E+04	-9.93318E-01
1.08000E+03 6.75723E-01 1.60933E+01 -1.06064E+04 -1.18231E+00				-1.06064E+04	-1.18231E+00
1.11000E+03 9.60443E-01 4.06358E+Q1 -9.78751E+03 -1.39187E+00		·		-9.78751E+03	-1.39187E+00
1.14000E+03 1.36151E+00 7.54650E+01 -8.09112E+03 -1.62026E+00		1111111		-8.09112E+03	-1.62026E+00
1.17000E+03 1.97805E+00 1.25559E+02 -5.14513E+03 -1.86392E+00					-1.86392E+00
1.20000E+03 6.31804E+00 .00000E+00 -1.24174E-10 -2.11667E+00			.00000E+00	-1.24174E-10	-2.11667E+00

To tabulate foundation pressures for a given interval: input $X-\min$, $X-\max$, and number of increments(input 0,0,0 to STOP) 0.,0.,0

Case III -- Concentrated End Couples

4. Concentrated end couples of 500 kips/ft is applied at end of the beam as shown in the following example.

INPUT: Problem title (for echo check put S in column one) Sample problem for WES with moment on ends (kips,inches)

SELECT AN OPTION: 1 = Plane strain, 2 = Plane stress

INPUT: Young's modulus and Poisson's ratio for the half plane 300...2

INPUT: The length, the depth, and Young's modulus for the beam 1201.,120.,3000.

To define the foundation interaction points select: X-min, X-max, the number of evenly spaced support points and the width of the support in the direction of the beam axis 0.,1200.,41,1.

For the external beam loading input: The number of concentrated loads and the number of linearly varying ramp loads 0.2

INPUT: The starting magnitude, starting position, end magnitude, and end position for each ramp load +40000.,0.,-40000.,3.
-40000.,1197.,40000.,1200.

Is the foundation flexibility matrix to be printed? (Y/N) \boldsymbol{n}

Is the foundation stiffness matrix to be printed? (Y/N) $\ensuremath{\text{n}}$

*** FOUNDATION FORCES AND DISPLACEMENTS ***

Index	Position	Reaction	Displacement
		Force	in beam
1	.00000E+00	-1.20259E+02	2.03478E+00
	3.00000E+01	-5.65107E+01	1.35781E+00
	6.00000E+01	-2.42488E+01	7.97552E-01
4 9	9.00000E+01	-5.26440E+00	3.43477E-01
	1.20000E+02	6.30331E+00	-1.67818E-02
6	1.50000E+02	1.32117E+01	-2.95995E-01
	1.80000E+02	1.70327E+01	-5.06591E-01
	2.10000E+02	1.87678E+01	-6.60201E-01
9 2	2.40000E+02	1.90987E+01	-7.67416E-01
	2.70000E+02	1.85073E+01	-8.37669E-01
	3.00000E+02	1.73414E+01	-8.79207E-01
	3.30000E+02	1.58539E+01	-8.99128E-01
	3.60000E+02	1.42289E+01	-9.03448E-01
	3.90000E+02	1.25997E+01	-8.97195E-01
	4.20000E+02	1.10616E+01	-8.84506E-01
	4.50000E+02	9.68173E+00	-8.68732E-01
	4.80000E+02	8.50656E+00	-8.52528E-01
	5.10000E+02	7.56735E+00	-8.37945E-01
	5.40000E+02	6.88443E+00	-8.26498E-01
	5.70000E+02	6.47026E+00	-8.19225E-01
	6.00000E+02	6.33150E+00	-8.16735E-01
	6.30000E+02	6.47026E+00	-8.19225E-01
_	6.60000E+02	6.88443E+00	-8.26498E-01
	6.90000E+02	7.56735E+00	-8.37945E-01
	7.20000E+02	8.50656E+00	-8.52528E-01
	7.50000E+02	9.68173E+00	-8.68732E-01
	7.80000E+02	1.10616E+01	-8.84506E-01
	8.10000E+02	1.25997E+01	-8.97195E-01
	8.40000E+02	1.42289E+01	-9.03448E-01
	8.70000E+02	1.58539E+01	-8.99128E-01
	9.00000E+02	1.73414E+01	-8.79207E-01
	9.30000E+02	1.85073E+01	-8.37669E-01
	9.60000E+02	1.90987E+01	-7.67416E-01
	9.90000E+02	1.87678E+01	-6.60201E-01
	1.02000E+03	1.70327E+01	-5.06591E-01
	1.05000E+03	1.32117E+01	-2.95995E-01
	1.08000E+03	6.30331E+00	-1.67818E-02
	1.11000E+03	-5.26440E+00	3.43477E-01
	1.14000E+03	-2.42488E+01	7.97552E-01
	1.17000E+03	-5.65107E+01	1.35781E+00
41	1.20000E+03	-1.20259E+02	2.03478E+00

Residual shear error at right end: .00000E+00 Residual moment error at right end: -4.51337E-11

To tabulate foundation pressures for a given interval: input X-min, X-max, and number of increments(input 0,0,0 to STOP) 0.,1200.,40

*** SUMMARY OF RESULTS FOR INTERFACE POINTS ***

Position	Interface Pressure	Shear	Moment	Displacement
.00000E+00	-8.01727E+00	.00000E+00	.00000E+00	2.03478E+00
3.00000E+01	-1.88369E+00	-1.48514E+02	5.70823E+04	1.35781E+00
6.00000E+01	-8.08294E-01	-1.88894E+02	5.19001E+04	7.97552E-01
9.00000E+01	-1.75480E-01	-2.03651E+02	4.59408E+04	3.43477E-01
1.20000E+02	2.10110E-01	-2.03131E+02	3.97957E+04	-1.67818E-02
1.50000E+02	4.40390E-01	-1.93374E+02	3.38222E+04	-2.95995E-01
1.80000E+02	5.67755E-01	-1.78252E+02	2.82335E+04	-5.06591E-01
2.10000E+02	6.25593E-01	-1.60351E+02	2.31479E+04	-6.60201E-01
2.40000E+02	6.36623E-01	-1.41418E+02	1.86201E+04	-7.67416E-01
2.70000E+02	6.16911E-01	-1.22615E+02	1.46619E+04	-8.37669E-01
3.00000E+02	5.78046E-01	-1.04691E+02	1.12566E+04	-8.79207E-01
3.30000E+02	5.28462E-01	-8.80932E+01	8.37046E+03	-8.99128E-01
3.60000E+02	4.74297E-01	-7.30518E+01	5.95938E+03	-9.03448E-01
3.90000E+02	4.19990E-01	-5.96375E+01	3.97515E+03	-8.97195E-01
4.20000E+02	3.68719E-01	-4.78069E+01	2.36925E+03	-8.84506E-01
4.50000E+02	3.22724E-01	-3.74352E+01	1.09580E+03	-8.68732E-01
4.80000E+02	2.83552E-01	-2.83411E+01	1.13559E+02	-8.52528E-01
5.10000E+02	2.52245E-01	-2.03041E+01	-6.12597E+02	-8.37945E-01
5.40000E+02	2.29481E-01	-1.30782E+01	-1.11077E+03	-8.26498E-01
5.70000E+02	2.15675E-01	-6.40088E+00	-1.40140E+03	-8.19225E-01
6.00000E+02	2.13075E-01 2.11050E-01	1.84741E-12	-1.49690E+03	-8.16735E-01
6.30000E+02	2.11030E-01 2.15675E-01	6.40088E+00	-1.49090E+03	-8.19225E-01
6.60000E+02	2.29481E-01	1.30782E+01	-1.11077E+03	-8.26498E-01
6.90000E+02	2.52245E-01	2.03041E+01	-6.12597E+02	-8.37945E-01
7.20000E+02	2.83552E-01	2.83411E+01	1.13559E+02	-8.52528E-01
7.50000E+02	3.22724E-01	3.74352E+01	1.09580E+03	-8.68732E-01
7.80000E+02	3.68719E-01	4.78069E+01	2.36925E+03	-8.84506E-01
8.10000E+02	4.19990E-01	5.96375E+01	3.97515E+03	-8.97195E-01
8.40000E+02	4.74297E-01	7.30518E+01	5.95938E+03	-9.03448E-01
8.70000E+02	5.28462E-01	8.80932E+01	8.37046E+03	-8.99128E-01
9.00000E+02	5.78046E-01	1.04691E+02	1.12566E+04	-8.79207E-01
9.30000E+02	6.16911E-01	1.22615E+02	1.46619E+04	-8.37669E-01
9.60000E+02	6.36623E-01	1.41418E+02	1.86201E+04	-7.67416E-01
9.90000E+02	6.25593E-01	1.60351E+02	2.31479E+04	-6.60201E-01
1.02000E+03	5,67755E-01	1.78252E+02	2.82335E+04	-5.06591E-01
1.05000E+03	4.40390E-01	1.93374E+02	3.38222E+04	-2.95995E-01
1.08000E+03	2.10110E-01	2.03131E+02	3.97957E+04	-1.67818E-02
1.11000E+03	-1.75480E-01	2.03651E+02	4.59408E+04	3.43477E-01
1.14000E+03	-8.08294E-01	1.88894E+02	5.19001E+04	7.97552E-01
1.17000E+03	-1.88369E+00	1.48514E+02	5.70823E+04	1.35781E+00
1.20000E+03	-8.01727E+00	3.60766E-12	-9.96564E-10	2.03478E+00

To tabulate foundation pressures for a given interval: input X-min, X-max, and number of increments(input 0,0,0 to STOP) 0.,0.,0

5. The computer codes for Program I sample problems are shown on the following pages.

```
1:SSSSSSS MAIN
 2:C
 3:C
         Written for:
                        The U.S. Corps of Engineers
 4:C
                        H.B. Wilson and L.H. Turcotte
 5:C
                        University of Alabama at Tuscaloosa
 6:C
         Date:
                         September 1983
 7:C
         IMPLICIT REAL*8
                            (A-H, O-Z)
 8:
9:
         CHARACTER*1
                            TITL(80), ANS
10:
         LOGICAL
                            ECHO
11:
         COMMON / BEAM
                          / BLEN, XFIX, NF, F(53), XF(53), NP, P(53),
12:
                            XP(53),NR,WL(53),XL(53),WR(53),XR(53),
13:
                            EIB
14:
         COMMON / QTOTAL / XT(53), IFRST, D, DH
15:
         COMMON / HAFPLA / EHAPL, POIHPL, TWOGHP, CAPA
16:
         COMMON / EQNS
                          / NFMAX,AHP(53,53),ABM(53,53),B(53)
17:
         COMMON / EQNS2 / ATOTL(53,53), IPIVOT(53), SCAL(53), ISTATE
18:
         COMMON / EQNS3 / AHPINV(53,53), DBM(53), SMALL, NODEI
19:
         DATA
                            PI/ 3.141592653589793D0 /
20:
         DATA
                            IRD, IPRT / 0, 0 /
21:C
22:
         NFMAX = 53
23:
         ECHO = .FALSE.
24:
         IFRST = 1
25:C
26:
         WRITE (IPRT.10)
        FORMAT(//.5X.'*** FOUNDATION INTERACTION BETWEEN A BEAM ****
27:
                 /,5X,'***
                                  AND AN ELASTIC HALF PLANE
28:
29:
         WRITE (IPRT, 20)
     20 FORMAT(/,' INPUT: Problem title (for echo check put $ in column ',
30:
31:
                  one)')
         READ (ICRT, 30) (TITL(I), I = 1,80)
32:
33:
     30 FORMAT(80A1)
         IF (TITL(1) .EQ. '$') ECHO = .TRUE.
34:
35:
         IF (ECHO) WRITE (IPRT, 40) (TITL(I), I = 2,80)
     40 FORMAT(1X,80A1)
36:
37:
         WRITE (IPRT, 50)
38:
     50 FORMAT(/,' SELECT AN OPTION: 1 = Plane strain, 2 = Plane stress')
         READ (ICRT,*) ISTATE
39:
40:
         IF (ECHO) WRITE (IPRT,*) ISTATE
41:
         WRITE (IPRT, 60)
42:
     60 FORMAT(/,' INPUT: Young''s modulus and Poisson''s ratio for ',
43:
                  'the half plane')
44:
         READ (ICRT,*) EHAPL, POIHPL
45:
         IF (ECHO) WRITE (IPRT,*) EHAPL, POIHPL
46:C...Define elastic constants
         TWOGHP = EHAPL / (1.DO + POIHPL)
47:
48:C...Plane strain
         IF (ISTATE .EQ. 1) CAPA = 3.DO - 4.DO * POIHPL
49:
50:C...Plane stress
         IF (ISTATE .EQ. 2) CAPA = (3.DO - POIHPL) / (1.DO + POIHPL)
51:
```

```
SHRMOD = EHAPL * .5DO / (1.DO + POIHPL)
 52:
 53:
          WRITE (IPRT,70)
      70 FORMAT(/,' INPUT: The length, the depth, and Young''s modulus ',
 54:
                    for the beam')
 55:
56:
          READ (ICRT,*) BLEN, BDEP, EBEAM
57:
          IF (ECHO) WRITE (IPRT.*) BLEN. BDEP. EBEAM
          EIB = EBEAM * (BDEP ** 3) / 12.00
58:
59:
          XFIX = 1.1DO * BLEN
          WRITE (IPRT,80)
60:
     80 FORMAT(/,' To define the foundation interaction points select:
61:
                  'X-min, X-max,',/,' the number of evenly spaced support',
 62:
63:
                    points',/,' and the width of the support in the ',
64:
                  'direction of the beam axis')
65:
         READ (ICRT,*) XMIN, XMAX, NF, WDTH
          IF (ECHO) WRITE (IPRT,*) XMIN,XMAX,NF,WDTH
67:
          IF (NF .LE. NFMAX-2) GO TO 100
68:
          WRITE (IPRT,90) NFMAX-2
69:
     90 FORMAT(/,' Maximum number of support points = ',I3,
70:
                     PROGRAM TERMINATED')
71:
          STOP ' '
72: 100 DF = (XMAX - XMIN) / (NF - 1)
          HAFWID = 0.5DO * WDTH
73:
          ALP = (1.DO + CAPA) / (8.DO * HAFWID * SHRMOD * PI)
74:
75:
          DO 110 I = 1.NF
76: 110 XF(I) = XMIN + DF * (I - 1)
          IF (BLEN .EQ. 0.0D0) GO TO 180
77:
          WRITE (IPRT, 120)
78:
79: 120 FORMAT(/,' For the external beam loading input: The ',/,
80:
                     ' number of concentrated loads and the number of ',/,
        &
81:
                     ' linearly varying ramp loads ')
         READ (ICRT,*) NP,NR
82:
83:
          IF (ECHO) WRITE (IPRT,*) NP, NR
84:
          IF (NP .EQ. 0) GO TO 150
          WRITE (IPRT, 130)
85:
86: 130 FORMAT(/,' INPUT: The magnitude and position of each applied ',
87:
                  'force')
88:
          DO 140 I = 1,NP
89:
          READ (ICRT,*) P(I),XP(I)
90:
          IF (ECHO) WRITE (IPRT,*) P(I),XP(I)
91: 140 CONTINUE
 92: 150 IF (NR .EQ. 0) GO TO 180
93:
          WRITE (IPRT, 160)
 94: 160 FORMAT(/,' INPUT: The starting magnitude, starting ',
                   position, end magnitude,',/,' and end position for ',
 95:
                  'each ramp load')
 96:
97:
          DO 170 I = 1.NR
98:
          READ (ICRT,*) WL(I),XL(I),WR(I),XR(I)
99:
          IF (ECHO) WRITE (IPRT,*) WL(I),XL(I),WR(I),XR(I)
100: 170 CONTINUE
101:C...Form equations and solve for the foundation equations
102: 180 CALL SOLEON(HAFWID, ALP)
103:
          WRITE (IPRT, 190)
104: 190 FORMAT(/,' Is the foundation flexibility matrix to be',
                    printed? (Y/N)')
```

```
READ (ICRT, 30) ANS
106:
          IF (ECHO) WRITE (IPRT,*) ANS
107:
          IF (ANS .EQ. 'N' .OR. ANS .EQ. 'n') GO TO 230
108:
109:
          WRITE (IPRT, 200)
110: 200
          FORMAT(/,' The Foundation Flexibility Matrix is:',/)
          DO 210 I = 1,NF
111:
112: 210
          WRITE (IPRT, 220) (AHP(I,J), J = 1.NF)
113: 220
         FORMAT(1X,6(1PE12.4),/,(13X,5(1PE12.4)))
114: 230 WRITE (IPRT, 240)
115: 240 FORMAT(/,' Is the foundation stiffness matrix to be printed? ',
116:
                   '(Y/N)')
          READ (ICRT, 30) ANS
117:
          IF (ECHO) WRITE (IPRT,*)
118:
                                     ANS
119:
          IF (ANS .EQ. 'N' .OR. ANS .EQ. 'n') GO TO 290
          CALL INVERT(AHP, NF, NFMAX, IPIVOT, SCAL, IFLAG, AHPINV)
120:
121:
          IF (IFLAG .NE. 2) GO TO 260
122:
          WRITE (IPRT, 250)
123: 250 FORMAT(/,' The Foundation Flexibility Matrix is Singular ',
                  /,' EXECUTION TERMINATED',/)
124:
          STOP '
125:
126: 260 WRITE (IPRT, 270)
127: 270 FORMAT(/,' The Foundation Stiffness Matrix is:',/)
128:
          DO 280 I = 1.NF
129: 280
          WRITE (IPRT, 220)
                            (AHPINV(I,J),J = 1,NF)
130: 290
          CONTINUE
          WRITE (IPRT, 300)
131:
132: 300 FORMAT(/,3X,' *** FOUNDATION FORCES AND DISPLACEMENTS ****
133:
                 //,' Index',2X,'Position',8X,'Reaction',6X,
134:
                 'Displacement',/,24X,'Force',11X,'in beam')
135:
          IF (SMALL .NE. O.DO) WRITE(IPRT,310)
136. 310 FORMAT(39X,'[relative]')
137:
          SUMF = Q(BLEN,1)
138:
          SUMM = Q(BLEN, 2)
139:
          DO 320 I=1,NF
140:
          SUMF = SUMF - F(I)
          SUMM = SUMM - F(I) * (BLEN-XF(I))
141:
142: 320
         WRITE(IPRT, 330) I,XF(I),-F(I),DBM(I)
143: 330 FORMAT(I4,3(1PE13.5,3X))
144:
          WRITE(IPRT, 340) SUMF, SUMM
145: 340 FORMAT(/,' Residual shear error at right end: ',1PE13.5,
                 /,' Residual moment error at right end: ',1PE13.5)
         æ
          IF (SMALL .NE. O.DO) WRITE(IPRT, 350) SMALL, NODEI
147:
148: 350 FORMAT(/, Displacements are relative to:
                                                          ',1PE13.5.
                   ' at node # ', I3)
149:
150:C...Compute shear and moment at interface points and output results
151: 360 WRITE(IPRT, 370)
152: 370 FORMAT(//,' To tabulate foundation pressures for a given '
153:
         &
                    'interval: input',/,' X-min, X-max, and number ',
154:
                    'of increments(input 0.0.0 to STOP)')
          READ(IRD,*) XXL,XXR,NSEG
155:
          IF (XXL .EQ. XXR) STOP ' '
156:
157:
          DX = (XXR-XXL) / DBLE(NSEG)
158:
          WRITE(IPRT, 380)
159: 380 FORMAT(/,15X,'*** SUMMARY OF RESULTS FOR INTERFACE POINTS ',
```

```
160:
                       '***', //,
161:
         &
                   4X, 'Position', 7X, 'Interface', 8X, 'Shear', 9X, 'Moment',
162:
                   6X, 'Displacement')
163:
          IF (SMALL .NE. O.ODO) WRITE(IPRT, 390)
164: 390 FORMAT(19X, 'Pressure', 36X, '[relative]')
          IF (SMALL .EQ. 0.0DO) WRITE(IPRT,400)
166: 400 FORMAT(19X, 'Pressure')
167:
          DO 410 I = 0.NSEG
          XX = XXL + I * DX
168:
          QT1 = QTOTL(XX,1)
169:
170:
          QT2 = QTOTL(XX,2)
          QT4 = QTOTL(XX,4)
171:
          FPR = FPRES(XX)
172:
173: 410 WRITE(IPRT, 420) XX, FPR, QT1, QT2, QT4
174: 420 FORMAT(1X,5(1PE13.5,2X))
175:
          GO TO 360
176:
          END
177:C$$$$$$$$ FS
178:
          REAL*8 FUNCTION FS(X,XBEGIN,NPOWR)
179:C...Singularity function
          IMPLICIT REAL*8 (A-H,O-Z)
181:
          FS = 0.0D0
182:
          D = X - XBEGIN
183:
          IF (D .LT. O.ODO) RETURN
184:
          IF (NPOWR .GT. 0) GO TO 10
185:
          FS = 1.D0
186:
          RETURN
     10 FS = D ** NPOWR
187:
188:
          RETURN
189:
          END
190:CSSSSSSSSS RAMP
          REAL*8 FUNCTION RAMP(X,XL,WL,XR,WR,EI,ID)
192: C... Load and deflection quantities for ramp load on a beam
          IMPLICIT REAL*8 (A-H,O-Z)
194:
          RAMP = 0.0D0
          DL = X - XL
195:
196:
          IF (DL .LT. O.ODO) RETURN
197:
          DR = X - XR
          S = (WR - WL) / (XR - XL)
198:
199:C
200:
          GO TO (10,20,30,40,50), ID+1
201:C
202:C...Load per unit length
203: 10 RAMP = WL * FS(X,XL,0) - WR * FS(X,XR,0)
204:
          IF (S .EQ. O.ODO) RETURN
205:
          RAMP = RAMP + S * (FS(X,XL,1) - FS(X,XR,1))
          RETURN
206:
207:C...Shear
208: 20 RAMP = WL * FS(X,XL,1) - WR * FS(X,XR,1)
          IF (S .EQ. O.ODO) RETURN
209:
210:
          RAMP = RAMP + 0.5DO * S * (FS(X,XL,2) - FS(X,XR,2))
211:
          RETURN
212:C...Moment
213: 30 RAMP = 0.5D0 * (WL * FS(X,XL,2) - WR * FS(X,XR,2))
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IF (S .EQ. O.ODO) RETURN
214:
          RAMP = RAMP + S / 6.0D0 * (FS(X,XL,3) - FS(X,XR,3))
215:
216:
          RETURN
217: C... Slope
218: 40 RAMP = (WL * FS(X,XL,3) - WR * FS(X,XR,3)) / 6.0D0
219:
          IF (S .EQ. 0.0D0) GO TO 60
220:
          RAMP = RAMP + S / 24.0D0 * (FS(X,XL,4) - FS(X,XR,4))
221:
          GO TO 60
222:C...Deflection
223: 50 RAMP = (WL * FS(X,XL,4) - WR * FS(X,XR,4)) / 24.0D0
224:
          IF (S .EQ. 0.0D0) GO TO 60
          RAMP = RAMP + S / 120.0D0 * (FS(X,XL,5) - FS(X,XR,5))
226:
     60 \text{ RAMP} = \text{RAMP} / \text{EI}
227:
          RETURN
228:
          END
229:CSSSSSSSSS FUNIT
          REAL*8 FUNCTION FUNIT(X, POSITN, EI, ID)
231: C... Load and deflection quantities for concentrated load on a beam
          IMPLICIT REAL*8 (A-H,O-Z)
232:
233:
          FUNIT = 0.0D0
234:
          IF (ID .EQ. O) RETURN
235:
          D = X - POSITN
236:
          IF (D .LT. O.ODO) RETURN
237:
          GO TO (10,20,30,40),ID
238:C...Shear
239: 10 FUNIT = FS(X, POSITN, 0)
240:
          RETURN
241:C...Moment
242: 20 FUNIT = FS(X, POSITN, 1)
243:
          RETURN
244:C...Slope
245: 30 FUNIT = FS(X, POSITN, 2) * 0.5D0 / EI
246:
          RETURN
247:C...Deflection
248: 40 FUNIT = FS(X, POSITN, 3) / (6.0D0 * EI)
249:
          RETURN
250:
          END
251:C$$$$$$$$$$
252:
          REAL*8 FUNCTION Q(X,ID)
253:C...Loading and deflection contribution of external concentrated
254:C...and ramp loads
          IMPLICIT REAL*8 (A-H,O-Z)
255:
256:
          COMMON / BEAM
                           / BLEN, XFIX, NF, F(53), XF(53), NP, P(53),
257:
         &
                             XP(53),NR,WL(53),XL(53),WR(53),XR(53),
258:
259:
          Q = 0.0D0
          IF (NP .EQ. 0)
                          GO TO 20
260:
          DO 10 J = 1,NP
261:
262:
     10 Q = Q + P(J) * FUNIT(X,XP(J),EIB,ID)
263:
      20 IF (NR .EQ. 0) RETURN
264:
          DO 30 J = 1.NR
265:
          Q = Q + RAMP(X,XL(J),WL(J),XR(J),WR(J),EIB,ID)
266:
          RETURN
267:
          END
```

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268:CSSSSSSSSS QTOTL
          REAL*8 FUNCTION QTOTL(X,ID)
269:
270: C... Total loading and deflection quantities
          IMPLICIT REAL*8 (A-H,O-Z)
                           / BLEN, XFIX, NF, F(53), XF(53), NP, P(53),
272:
          COMMON / BEAM
273:
                             XP(53),NR,WL(53),XL(53),WR(53),XR(53),
274:
                             EIB
275:
          COMMON / QTOTAL / XT(53), IFRST, D, DH
276:
          COMMON / EQNS3 / AHPINV(53,53), DBM(53), SMALL, NODEI
277: C... Initialize array XT on first entry
278:
          IF (IFRST .EQ. 0) GO TO 20
          IFRST = 0
279:
280:
          N1 = NF - 1
281:
          XT(1) = XF(1)
282:
          XT(NF+1) = XF(NF)
          D = (XF(NF)-XF(1))/DBLE(N1)
283:
284:
          DH = 0.5D0 * D
          DO 10 I = 2, NF
285:
286:
      10 XT(I) = XF(I) - DH
287:C...
288: 20
          IF (ID .GT. 2) GO TO 50
289:
          XLT = XT(1)
290:
          XRT = XT(2)
291:
          WT = 2.0DO * F(1)
292:
          QT = RAMP(X,XLT,WT,XRT,WT,EIB,ID)
293:
          DO 30 I = 2, N1
294:
          XLT = XT(I)
295:
          IF (XLT .GE. X) GO TO 40
296:
          XRT = XT(I+1)
297:
          WT = F(I)
298:
      30 QT = QT + RAMP(X, XLT, WT, XRT, WT, EIB, ID)
299:
          XLT = XT(NF)
300:
          IF (X .LE. XLT) GO TO 40
301:
          XRT = XT(NF+1)
          WT = 2.0D0 * F(NF)
302:
303:
          QT = QT + RAMP(X,XLT,WT,XRT,WT,EIB,ID)
304:
      40 QTOTL = Q(X,ID) - QT / D
305:
          RETURN
306:C...Interpolate lineraly for slope or deflection
307:
     50 ID2 = ID - 2
308:
          QTOTL = FLNTRP(X, XF, DBM, NF, ID2)
309:
          RETURN
310:
          END
311:CSSSSSSSSS FPRES
312:
          REAL*8 FUNCTION FPRES(X)
313:C...Function to determine piecewise constant foundation pressure
314:
          IMPLICIT REAL*8 (A-H,O-Z)
315:
          COMMON / QTOTAL / XT(53), IFRST, D, DH
316:
          COMMON / BEAM
                           / BLEN, XFIX, NF, F(53), XF(53), NP, P(53),
317:
         &
                             XP(53),NR,WL(53),XL(53),WR(53),XR(53),
318:
                             EIB
          IF (X .GT. XT(2)) GO TO 10
319:
320: C... Pressure in short left segment
          FPRES = 2.DO * F(1)
321:
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GO TO 50
322:
323: 10 IF (X .LT. XT(NF)) GO TO 20
324:C...Pressure in right short segment
          FPRES = 2.DO * F(NF)
325:
          GO TO 50
326:
327:C...Determine segment containing X
328: 20 DO 30 I = 3.NF
          ISV = I
329:
330:
          IF (XT(I) .GE. X) GO TO 40
331:
     30 CONTINUE
332:
     40
          FPRES = F(ISV-1)
333: 50 FPRES = -FPRES / D
334:
          RETURN
335:
          END
336:CSSSSSSSSS FACTR
337:
          SUBROUTINE FACTR (A,N,NMAX,IPIVOT,SCAL,IFLAG)
338:C...Perform triangular factorization of matrix a
339: C...using scaled row pivoting
            IFLAG = 1 means normal return
340:C...
341:C...
                  = 2 means matrix is singular
342:
          IMPLICIT REAL*8 (A-H,O-Z)
343:
          DIMENSION A(NMAX,N), IPIVOT(N), SCAL(N)
344:
          IFLAG = 1
345:C...Initialize IPIVOT and SCAL
346:
          DO 20 I = 1.N
347:
          IPIVOT(I) = I
348:
          ROWMAX = 0.0DO
349:
          DO 10 J = 1.N
350:
       10 ROWMAX = DMAX1(ROWMAX, DABS(A(I,J)))
351:
          IF (ROWMAX.EQ.O.ODO) GO TO 50
       20 \text{ SCAL}(I) = \text{ROWMAX}
352:
353: C... Perform Gauss reduction
          NM1 = N-1
354:
355:
          IF (NM1.EQ.O) RETURN
356:
          DO 40 K = 1.NM1
357:
          J = K
358:
          KP1 = K+1
359:
          IP = IPIVOT(K)
360:
          COLMAX = DABS(A(IP,K))/SCAL(IP)
361:
          DO 30 I = KP1.N
362:
          IP = IPIVOT(I)
363:
          AWIKOV = DABS(A(IP,K))/SCAL(IP)
364:
          IF (AWIKOV.LE.COLMAX) GO TO 30
365:
          COLMAX = AWIKOV
366:
          J = I
367:
       30 CONTINUE
368:
          IF (COLMAX.EQ.O.ODO) GO TO 50
369:
          IPK = IPIVOT(J)
370:
          IPIVOT(J) = IPIVOT(K)
371:
          IPIVOT(K) = IPK
372:
          DO 40 I = KP1.N
          IP = IPIVOT(I)
373:
374:
          A(IP,K) = A(IP,K)/A(IPK,K)
375:
          RATIO = -A(IP,K)
```

```
376:
          DO 40 J = KP1,N
       40 A(IP,J) = RATIO*A(IPK,J)+A(IP,J)
377:
378:
          IF (A(IP,N).EQ.O.ODO) GO TO 50
379:
          RETURN
380:
       50 \text{ IFLAG} = 2
          RETURN
381:
382:
          END
383:CS$$$$$$$$$ BAKSUB
384:
          SUBROUTINE BAKSUB (A,N,NMAX,B,IPIVOT,X)
385:C...Solve simultaneous equations AX = B where matrix A has
386: C... been subjected to factorization by SUBROUTINE FACTR
          IMPLICIT REAL*8 (A-H,O-Z)
388:
          DIMENSION A(NMAX,N), B(N), IPIVOT(N), X(N)
389:
          IF (N.GT.1) GO TO 10
390:
          X(1) = B(1)/A(1,1)
391:
          RETURN
392:
       10 IP = IPIVOT(1)
393:
          X(1) = B(IP)
394:
          DO 30 K = 2,N
395:
          IP = IPIVOT(K)
396:
          KM1 = K-1
          SUM = 0.0D0
397:
398:
          DO 20 J = 1, KM1
       20 SUM = A(IP,J)*X(J)+SUM
399:
400:
       30 X(K) = B(IP)-SUM
401:
          X(N) = X(N)/A(IP,N)
          K = N
402:
403:
          DO 50 NP1MK = 2.N
404:
          KP1 = K
405:
          K = K-1
406:
          IP = IPIVOT(K)
407:
          SUM = 0.0D0
408:
          DO 40 J = KP1,N
409:
       40 SUM = A(IP,J)*X(J)+SUM
410:
       50 X(K) = (X(K)-SUM)/A(IP,K)
411:
          RETURN
412:
          END
413:CSSSSSSSSS INVERT
          SUBROUTINE INVERT(A,N,NMAX,IPIVOT,SCAL,IFLAG,AINV)
415:C...IFLAG indicates singular matrix
          IMPLICIT REAL*8 (A-H,O-Z)
416:
417:
          DIMENSION A(NMAX,1),AINV(NMAX,1),IPIVOT(1),SCAL(1)
418:
          CALL FACTR(A,N,NMAX,IPIVOT,SCAL,IFLAG) .
419:
          IF (IFLAG .EQ. 2) RETURN
420:
          DO 10 J = 1,N
421:
      10 \quad SCAL(J) = 0.0D0
422:
          DO 20 I = 1,N
423:
          SCAL(I) = 1.0D0
424:
          CALL BAKSUB(A,N,NMAX,SCAL,IPIVOT,AINV(1,I))
425:
          SCAL(I) = 0.0D0
426:
      20
          CONTINUE
427:
          RETURN
428:
          END
429:CSSSSSSSSS SOLEQN
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430:
          SUBROUTINE SOLEON(HAFWID, ALP)
431:C...This subroutine forms the flexibility matrices for a beam
432:C...and for a half plane subjected to several concentrated
433:C...loads. These matrices are then combined and the resulting
434:C...simultaneous equations are solved for the interaction forces.
          IMPLICIT REAL*8 (A-H,O-Z)
436:
          COMMON / BEAM
                          / BLEN, XFIX, NF, F(53), XF(53), NP, P(53),
437:
                            XP(53),NR,WL(53),XL(53),WR(53),XR(53),
438:
439:
          COMMON / HAFPLA / EHAPL, POIHPL, TWOGHP, CAPA
440:
          COMMON / EQNS
                         / NFMAX,AHP(53,53),ABM(53,53),B(53)
441:
          COMMON / EQNS2 / ATOTL(53,53), IPIVOT(53), SCAL(53), ISTATE
442:
          COMMON / EQNS3 / AHPINV(53,53), DBM(53), SMALL, NODEI
443:
          DATA
                            IPRT/ 0 /
444:
          NF1 = NF + 1
445:
          NF2 = NF + 2
446:C...Define some scaling factors
          SCAL1 = BLEN ** 3 / EIB
447:
          SCAL2 = BLEN ** 2 / EIB
448:
449:C...Form influence for half plane
450:
          DO 10 I = 1.NF
451:
          DO 10 J = 1,NF
452: 10 AHP(I,J) = FORIGN(XF(I)-XF(J), HAFWID, XFIX, ALP)
453:C...Form beam influence if requested
454:
          IF (BLEN .EQ. O.ODO) RETURN
455:
          DO 20 I = 1,NF
456:
          DO 20 J = 1,NF
     20 ABM(I,J) = FS(XF(I),XF(J),3) / (6.0D0 * EIB)
457:
458: C... Form right side vector for applied beam loads
          DO 30 I = 1.NF
460: 30 B(I)=Q(XF(I),4)
461:C...Form combined influence
462:
          DO 40 I = 1.NF
463:
          DO 40 J = 1,NF
464: 40 ATOTL(I,J) = AHP(I,J) + ABM(I,J)
465:C...Define additional static equilibrium equations
466:
          DO 50 J = 1.NF
467:
          ATOTL(J,NF1) = -SCAL1
          ATOTL(J,NF2) = -XF(J) * SCAL2
468:
469:
          ATOTL(NF1,J) = 1.0D0
470: 50 ATOTL(NF2,J) = BLEN - XF(J)
471:
          ATOTL(NF1,NF1) = 0.0D0
472:
          ATOTL(NF1,NF2) = 0.0D0
473:
          ATOTL(NF2,NF1) = 0.0D0
474:
          ATOTL(NF2,NF2) = 0.0D0
475:
          B(NF1) = Q(BLEN,1)
          B(NF2) = Q(BLEN, 2)
477:C...Solve for the interaction forces
          CALL FACTR(ATOTL, NF2, NFMAX, IPIVOT, SCAL, IFLAG)
479:
          IF (IFLAG .NE. 2) GO TO 70
480:
          WRITE (IPRT, 60)
      60 FORMAT(/,' COMBINED STIFFNESS MATRIX IS SINGULAR',/,
481:
482:
                      ' EXECUTION IS TERMINATED')
          STOP ' '
483:
```

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484:C...Compute the forces at interaction points
485: 70 CALL BAKSUB(ATOTL, NF2, NFMAX, B, IPIVOT, F)
486:C...Compute left end slope and displacement
          F(NF2) = F(NF2) * SCAL2
487:
          F(NF1) = F(NF1) * SCAL1
488:
489: C... Compute the displacements at interaction points
490:
          DO 90 I = 1,NF
          DBMI = B(I) + F(NF1) + F(NF2) * XF(I)
491:
492:
          DO 80 J = 1,NF
493: 80 DBMI = DBMI - ABM(I,J) * F(J)
494: 90 DBM(I) = DBMI
495:C...Find smallest and largest displacement in beam
496:
          BIG = +1.D20
497:
          SMALL = -1.D20
498:
          DO 100 I = 1.NF
499:
          IF( DBM(I) .GT. SMALL ) THEN
500:
               SMALL = DBM(I)
501:
               NODEI = I
502:
          ENDIF
503:
          IF( DBM(I) .LT. BIG ) THEN
504:
               BIG = DBM(I)
          ENDIF
505:
506: 100 CONTINUE
507:C...Make all displacements relative to SMALL=0 if displacements do
508:C...not change sign
509:
          IF( BIG*SMALL .LT. O.DO ) SMALL ≈ O.DO
510:C...Shift all displacements by value of SMALL
          DO 110 I =1.NF
512: 110 DBM(I) = DBM(I) - SMALL
          RETURN
513:
514:
          END
515:C$$$$$$$$$$ FORIGN
         REAL*8 FUNCTION FORIGN(X,A,XFIX,ALP)
517:C...Force at origin
518: C... A is the half width
          IMPLICIT REAL*8 (A-H,O-Z)
520:
          F(T) = ALP*((T-A)*DLOG(DABS(T-A))-(T+A)*DLOG(DABS(T+A)))
521:
          Z = DABS(X)
522:
          FORIGN = F(Z)-F(XFIX)
523:
          RETURN
524:
          END
525:CSSSSSSSSS FLNTRP
          REAL*8 FUNCTION FLNTRP(X,U,V,NDPTS,ITYP).
527:C...Function for linear interpolation on tabular data
528:C...ITYP=1 gives slope, ITYP=2 gives function value
          IMPLICIT REAL*8 (A-H,O-Z)
529:
530:
          DIMENSION U(1), V(1)
531:
          IF (X .GT. U(2)) GO TG 10
532:
          K2 = 2
533:
          GO TO 40
534: 10
          IF (X .LT. U(NDPTS-1)) GO TO 20
535:
          K2 = NDPTS
536:
          GO TO 40
537: 20
          K2 = 2
```

```
538: 30
          K2 = K2 + 1
539:
          IF (X .GT. U(K2)) GO TO 30
540: 40
          K1 = K2 - 1
541:
          IF (DABS(U(K1)-U(K2)) .GT. 0.0D0) GO TO 50
542:
          FLNTRP = 0.5D0 * V(K1) + 0.5D0 * V(K2)
543:
          RETURN
544: 50
          IF (ITYP.EQ.1) FLNTRP = (V(K2)-V(K1)) / (U(K2)-U(K1))
545:
          IF (ITYP.EQ.2)
546:
         & FLNTRP = V(K1) + (V(K2)-V(K1)) * (X-U(K1)) / (U(K2)-U(K1))
547:
          RETURN
548:
          END
```

APPENDIX B: PROGRAM II--TWO-DIMENSIONAL STIFFNESS MATRIX GENERATION FOR A HALF-PLANE OR A SYMMETRICALLY LOADED HALF-SPACE

1. Program II generalizes the flexibility and stiffness matrix formulation used in Program I. For problems considering only vertical loads, stiffness and flexibility matrices can be computed for the half-plane in plane strain or plane stress. An analogous formulation for a three-dimensional beam having a finite width and resting on a half-space is also included. A more significant addition to earlier results is that stiffness matrices for the half-plane subjected to surface loads involving horizontal components, vertical components, and couples can be obtained. Numerical results are shown for a simple case involving four support points, with the computer code immediately following.

2. On the following pages is a two-dimensional plane-strain analysis of a sample problem with vertical surface loads.

```
Is data to be echo printed? (Y/N)
N
Is the program explanation to be printed? (Y/N)
Y
```

This program determines the two-dimensional stiffness matrix for a series of loads applied along the X-axis on the surface of a half-plane or a half-space. Solutions based on two-dimensional plane strain, two-dimensional plane stress, or three-dimensional loading can be obtained. When a plane stress or plane-strain condition is considered, two types of loading can be applied 1) a series of vertical loads can be applied at a

- a series of vertical loads can be applied at a specified series of points,
- 2) a series of horizontal loads, vertical loads, and couples can be applied.

An influence function is used which computes the deflection at any position due to a unit load applied at the origin. For plane strain, a load of unity per unit of thickness is distributed uniformly between X=-.5*A and X=.5*A. For the three-dimensional loading, a unit load is distributed uniformly over a rectangular zone bounded by X=-.5*A, X=.5*A, Z=-.5*B, Z=.5*B where the Z direction is measured normal to the XY plane. The elements of the stiffness matrix depend on E (Young's modulus), NU (Poisson's ratio), the dimension parameters A and B, and the loading positions X(1),...,X(N). The elastic constants enter only from a multiplicative factor which is proportional to E for plane stress or proportional to E/(1.-NU**2) for both plane strain and three-dimensional loading.

```
Input values for Young's modulus and Poisson's ratio 3.E5,.2

Select the type of loading condition  
1 = Plane strain  
2 = Plane stress  
3 = Three dimensional loading  
1

For a plane strain or plane stress condition, identify the the type of surface loading  
1 = vertical surfaces loads only  
2 = horizontal loads, vertical loads, and couples 1
```

How are the loading positions specified?

appropriate proposed transfer appropriate to the second transfer transfer to the second transfer transfer to the second transfer t

```
1 = Take N evenly spaced positions between
      XMIN and XMAX
  2 = Position coordinates X(1),...,X(N) are
      input by the user
Input XMIN, XMAX and the number of
evenly spaced loading points
0.,100.,3
Loading positions are :
     X(I)
  1
        .0000
     50.0000
  3 100.0000
For a plane strain or plane-stress solution
input the loading zone width A (measured in the X direction)
and the position coordinate B at which zero displacement is imposed
.1,110.
Select compute option:
1 = compute both flexibility and stiffness matrices
2 = compute flexibility.matrix only
Select a print option:
 1 = print flexibility matrix only
  2 = print stiffness matrix only
  3 = print both matrices
The flexibility matrix elements on and
below the main diagonal are shown below:
  1.772E-05
  1.606E-06 1.772E-05
  1.942E-07 1.606E-06 1.772E-05
The stiffness matrix elements on and
below the main diagonal are shown below:
  5.692E+04
 -5.146E+03 5.738E+04
 -1.572E+02 -5.146E+03 5.692E+04
```

Problem analysis is completed

3. The solution for a second plane-strain two-dimensional problem with horizontal loads, vertical loads, and couples follows. Is data to be echo printed? (Y/N) Is the program explanation to be printed? (Y/N) Input values for Young's modulus and Poisson's ratio 3.E5..2 Select the type of loading condition 1 = Plane strain 2 = Plane stress 3 = Three dimensional loading For a plane strain or plane stress condition, identify the the type of surface loading 1 = vertical surfaces loads only 2 = horizontal loads, vertical loads, and couples How are the loading positions specified? 1 = Take N evenly spaced positions between XMIN and XMAX 2 = Position coordinates X(1),...,X(N) are input by the user Input XMIN, XMAX and the number of evenly spaced loading points 0.,100.,3 Loading positions are : X(I).0000 1 50.0000 100.0000 for a plane strain or plane stress solution input the loading zone width A (measured in the X direction) and the position coordinate B at which zero displacement is imposed .1,110. Select compute option: 1 = compute both flexibility and stiffness matrices 2 = compute flexibility matrix only

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```
Select a print option:
 l = print flexibility matrix only
 2 = print stiffness matrix only
 3 = print both matrices
The flexibility matrix elements on and
below the main diagonal are shown below:
 1.772E-05
   .000E+00 1.772E-05
 2.400E-05
            .000E+00 9.600E-04
                       .000E+00 1.772E-05
 1.606E-06 -1.200E-06
 1.200E-06 1.606E-06 4.074E-08
                                 .000E+00 1.772E-05
  .000E+00 -4.074E-08 -9.600E-10 2.400E-05
                                              .000E+00 9.600E-04
 1.942E-07 -1.200E-06
                       .000E+00 1.606E-06 -1.200E-06
                                                        .000E+00
            1.772E-05
 1.200E-06 1.942E-07 2.037E-08 1.200E-06 1.606E-06 4.074E-08
             .000E+00 1.772E-05
   .000E+00 -2.037E-08 -2.400E-10
                                  .000E+00 -4.074E-08 -9.600E-10
            2.400E-05
                       .000E+00 9.600E-04
The stiffness matrix elements on and
below the main diagonal are shown below:
 5.942E+04
 -1.237E+01 5.738E+04
 -1.485E+03 5.278E-01 1.079E+03
 -5.303E+03 3.739E+03 1.327E+02 5.994E+04
 -3.749E+03 -4.937E+03 9.137E+01 5.073E-14 5.785E+04
 1.327E+02 -9.102E+01 -3.319E+00 -1.498E+03 1.801E-15 1.079E+03
 -4.399E+02 3.324E+03 1.084E+01 -5.303E+03 3.749E+03 1.327E+02
            5.942E+04
 -3.324E+03 -4.334E+02 8.210E+01 -3.739E+03 -4.937E+03 9.102E+01
            1.237E+01 5.738E+04
  1.084E+01 -8.210E+01 -2.668E-01 1.327E+02 -9.137E+01 -3.319E+00
           -1.485E+03 -5.278E-01 1.079E+03
```

Problem analysis is completed

4. The computer code for Program II sample problems is printed on the following pages.

```
1:CSSSSSSSSS MAIN
 2:C...
             Program to determine foundation stiffness matrix
 3:C...
 4:C...
 5:C...Written by: Howard Wilson and Louis Turcotte
 6:C...
                    University of Alabama at Tuscaloosa
 7:C...For:
                    U.S. Army Corps of Engineers, WES
 8:C...Date:
                    September 1983
 9:C...
10:
         IMPLICIT REAL*8 (A-H,O-Z)
11:
         LOGICAL
                         PRNT
12:
         CHARACTER*1
                         ANS
13:
         COMMON /ONE/
                         FLEX(60.60).STIF(60.60)
14:
         COMMON /TWO/
                         X(60),STOR(60),ISTOR(60),FLXSTO(60,60)
15:
         DATA
                         IRD/O/, ICRT/O/, PRNT/.FALSE./, NMAX/60/
16:
         WRITE(ICRT, 10)
     10 FORMAT(/,
17:
        &5X,'***
                  TWO-DIMENSIONAL STIFFNESS MATRIX GENERATION
18:
        &5X,'***
19:
                      FOR A HALF-PLANE OR A SYMMETRICALLY
        &5X,'***
20:
                               LOADED HALF-SPACE
21:
         WRITE(ICRT, 20)
22:
     20 FORMAT(/,' Is data to be echo printed? (Y/N)')
         READ(IRD, 30) ANS
23:
24:
     30 FORMAT(A1)
         IF (ANS .EQ. 'Y' .OR. ANS .EQ. 'y') PRNT=.TRUE.
25:
26:
         IF (PRNT) WRITE(ICRT, 40) ANS
27:
    40 FORMAT(1X,A1)
         WRITE(ICRT,50)
28:
29:
     50 FORMAT(/,' Is the program explanation to be printed? (Y/N)')
30:
         READ(IRD, 30) ANS
31:
         IF (PRNT) WRITE(ICRT, 40) ANS
32:
         IF (ANS .NE. 'Y' .AND. ANS .NE. 'y') GO TO 100
         WRITE(ICRT, 60)
33:
     60 FORMAT(/,
34:
35:
        &1X,' This program determines the two-dimensional stiffness',/,
36:
        &lX, 'matrix for a series of loads applied along the X-axis',/,
37:
        &1X, 'on the surface of a half-plane or a half-space.',/,
38:
        &lX,'Solutions based on two-dimensional plane strain, two-di-',/,
39:
        &lX, mensional plane stress, or three-dimensional loading can',/,
40:
        &lX, be obtained. When a plane stress or plane strain condition')
41:
         WRITE(ICRT, 70)
     70 FORMAT(
42:
43:
        &IX, 'is considered, two types of loading can be applied',/,
44:
        &lX,'l) a series of vertical loads can be applied at a',/,
45:
        &lX,' specified series of points,',/,
        &1X,'2) a series of horizontal loads, vertical loads, and',/,
46:
                couples can be applied.',/,
47:
        &1X,'An influence function is used which computes the')
48:
49:
         WRITE(ICRT,80)
     80 FORMAT(
50:
51:
        &lX, 'deflection at any position due to a unit load applied at',/,
```

```
52:
         &lX,'the origin. For plane strain, a load of unity per unit of',/,
        &1X, 'thickness is distributed uniformly between X=-.5*A and',/,
53:
54:
         &lX, 'X=.5*A. For the three-dimensional loading, a unit load is',/,
         &lX, 'distributed uniformly over a rectangular zone bounded by',/,
55:
             'X=-.5*A, X=.5*A, Z=-.5*B, Z=.5*B where the Z direction is',/,
56:
57:
         &lX, 'measured normal to the XY plane. The elements of the stiff-')
58:
          WRITE(ICRT, 90)
59:
     90 FORMAT(
         &lX, 'ness matrix depend on E (Young''s modulus), NU (Poisson''s',/,
60:
61:
         &lX, 'ratio', the dimension parameters A and B, and the loading',/,
62:
         &1X, 'positions X(1),...,X(N). The elastic constants enter only',/,
63:
         &1X, from a multiplicative factor which is proportional to E'./.
64:
         &lX, for plane stress or proportional to E/(1.-NU**2) for both',/,
65:
         &IX, 'plane strain and three-dimensional loading.')
66: 100
         WRITE(ICRT,110)
67: 110 FORMAT(/,' Input values for Young''s modulus',/,
         & ' and Poisson''s ratio')
68:
69:
          READ(IRD,*) E,POIS
70:
          IF (PRNT) WRITE(ICRT.120) E.POIS
71: 120 FORMAT(1X,1PE11.5,F7.4)
72:
          WRITE(ICRT, 130)
73: 130 FORMAT(/,' Select the type of loading condition',/,
              1 = Plane strain',/,
74:
              2 = Plane stress',/,
75:
         &'
76:
              3 = Three dimensional loading')
77:
          READ(IRD,*) ITYPE
78:
          IF (PRNT) WRITE(ICRT, 140) ITYPE
79: 140 FORMAT(1X, I2)
80:
          IF (ITYPE .EQ. 3) GO TO 160
81:
          WRITE(ICRT, 150)
82: 150 FORMAT(/,
83:
         &' For a plane strain or plane stress condition, identify the',/,
84:
         &' the type of surface loading ',/,
85:
              1 = vertical surfaces loads only',/,
86:
              2 = horizontal loads, vertical loads, and couples')
87:
          ILOAD = 0
          READ(IRD,*) ILOAD
88:
89:
          IF (PRNT) WRITE(ICRT, 140) ILOAD
90: 160 WRITE(ICRT,170)
          FORMAT(/,' How are the loading positions specified?',/,
91: 170
92:
              1 = Take N evenly spaced positions between',/,
                  XMIN and XMAX',/,
93:
         &'
94:
         &'
              2 = Position coordinates X(1),...,X(N) are',/,
95:
                  input by the user')
96:
          READ(IRD,*) IOPT
97:
          IF (PRNT) WRITE(ICRT, 140) IOPT
98:
          IF (IOPT .EQ. 2) GO TO 240
99:
          WRITE(ICRT, 180)
100: 180 FORMAT(/,' Input XMIN, XMAX and the number of',/,
101:
         &' evenly spaced loading points')
102:
          READ(IRD,*) XMIN,XMAX,N
103:
          IF (PRNT) WRITE(ICRT, 190) XMIN, XMAX, N
104: 190 FORMAT(1X,F10.4,',',F10.4,',',I3)
105:
          D = (XMAX - XMIN) / (N-1)
```

```
106:
          DO 200 I=1,N
107: 200
         X(I) = XMIN + D * (I-1)
108:
          WRITE(ICRT, 210)
109: 210
          FORMAT(/,' Loading positions are :',/,' I',5X,'X(I)')
110:
          DO 220 I=1,N
111: 220
         WRITE(ICRT, 230) I,X(I)
112: 230 FORMAT(1X, I3, F10.4)
113:
          GO TO 270
114: 240
         WRITE(ICRT, 250)
115: 250 FORMAT(/,' Input N and X(1),...,X(N)')
116:
          READ(IRD, *) N, (X(K), K=1, N)
117:
          IF (PRNT) WRITE(ICRT, 260) N, (X(K), K=1, N)
118: 260 FORMAT(1X, I3,/,
119:
         & 7(F10.4,',',F10.4,',',F10.4,',',F10.4,',',F10.4,',',F10.4,',))
120: 270 IF(ITYPE.EQ.3) GO TO 300
121:
          WRITE(ICRT, 280)
122: 280 FORMAT(/,' For a plane strain or plane stress solution',/,
123:
         &' input the loading zone width A (measured in the X direction)',
124:
         &/,' and the position coordinate B at which zero displacement',
125:
         &' is imposed')
126:
          READ(IRD,*) A,B
127:
          IF (PRNT) WRITE(ICRT, 290) A, B
128: 290 FORMAT(1X,F10.4,',',F10.4)
129:
          GO TO 315
130: 300 WRITE(ICRT, 310)
131: 310 FORMAT(/,' For the dimensions of loading input the',/,
132:
         &' X-width and the Z-width of the loading zone')
133:
          READ(IRD,*) A,B
          IF (PRNT) WRITE(ICRT, 290) A, B
134:
135: 315 WRITE(ICRT, 320)
136: 320 FORMAT(/, 'Select compute option:',/,
                    ' 1 = compute both flexibility and stiffness matrices',/,
137:
         &
                    ' 2 = compute flexibility matrix only')
138:
          READ(IRD,*) NOSTIF
139:
          IF (PRNT) WRITE(ICRT, 140) NOSTIF
140:
141:
          NOSTIF = NOSTIF - 1
142: 330
          IF (ILOAD .EQ. 1 .OR. ITYPE .EQ. 3 ) THEN
           CALL STFVLO(X,N,NMAX,E,POIS,A,B,ITYPE,STOR,ISTOR,IERROR,NOSTIF,
143:
144:
                       FLXSTO, FLEX, STIF)
145:
          ELSE
146:
           CALL STFHVM(X,N,NMAX,E,POIS,A,B,ITYPE,STOR,ISTOR,IERROR,NOSTIF,
147:
                       FLXSTO, FLEX, STIF)
148:
          ENDIF
149:
          IF (IERROR .NE. 1) GO TO 350
150:
          WRITE(ICRT, 340)
151: 340 FORMAT(/,' A SINGULAR FLEXIBILITY MATRIX WAS',/,
152:
         &' OBTAINED. EXECUTION IS TERMINATED',/)
153:
          STOP ' '
154: 350 WRITE(ICRT, 360)
155: 360 FORMAT(/,' Select a print option:',
                 /,'
156:
         &
                       1 = print flexibility matrix only',
157:
         &
                       2 = print stiffness matrix only',
158:
                        3 = print both matrices')
          READ(IRD,*) NOPT
159:
```

```
160:
          IF (PRNT) WRITE(ICRT, 140) NOPT
161:
162:
          IF (ILOAD .EQ. 2) NN = 3 * N
163:
          IF (NOPT .EQ. 2) GO TO 390
164:
          WRITE(ICRT, 370)
165: 370 FORMAT(/, The flexibility matrix elements on and',/,
166:
         &' below the main diagonal are shown below:')
167:
          DO 380 I=1,NN
168: 380 WRITE(ICRT, 420) (FLEX(I, K), K=1, I)
169: 390 IF (NOPT .EQ. 1) GO TO 430
170:
          WRITE(ICRT, 400)
171: 400 FORMAT(/,' The stiffness matrix elements on and ',/,
         &' below the main diagonal are shown below:')
172:
173:
          DO 410 I=1,NN
174: 410 WRITE(ICRT, 420) (STIF(I, K), K=1, I)
175: 420 FORMAT(1X,6(1PE11.3),/,12X,5(1PE11.3),/,12X,5(1PE11.3),/,
         & 12X,5(1PE11.3),/,12X,5(1PE11.3),/,12X,5(1PE11.3),/,
177:
         & 12X,5(1PE11.3),/,12X,5(1PE11.3),/,12X,5(1PE11.3))
178: 430 WRITE(ICRT, 440)
179: 440
          FORMAT(/,' Problem analysis is completed',/)
          STOP '
180:
181:
          END
182:CSSSSSSSSS STFVLO
          SUBROUTINE STFVLO(X,N,NMAX,E,POIS,A,B,ITYPE,STOR,ISTOR,NOSTIF,
183:
184:
                            IERROR,FLXSTO,FLEX,STIF)
185: C... Stiffness matrix and flexibility formulation for half-plane
186:C...or half-space subjected to vertical loads only.
187:C...
               = Young s modulus
188:C...E
189:C...POIS
               = Poisson's ratio.
               = the loading dimension parameters
190:C...A. B
191:C...X(1),...,X(N) are the load application positions
192:C...ITYPE = 1,2, or 3 for plane strain, plane stress, or
                 three-dimensional loading
194:C...NOSTIF = 1 to generate only flexibility matrix
                 O to generate both flexibility and stiffness matrix
196:C...FLEX
               = the flexibility matrix
               = the stiffness matrix obtained by inverting the flexibility
197:C...STIF
198:C...
                 matrix when NOSTIF = 0
199:C...IERROR = 1 for a normal return
200:C...
                 2 for flexibility matrix that is singular
               = working storage of length N or more
201:C...STOR
202:C...ISTOR = working storage of length N or more
203:C...STIF
               = the N by N roundation stiffness matrix which is the
204:C...
                 desired subroutine output
205:
          IMPLICIT REAL*8 (A-H,O-Z)
206:
                           X(N), FLEX(NMAX, N), STIF(NMAX, N), STOR(1), ISTOR(1),
          DIMENSION
207:
                           FLXSTO(NMAX,N)
208:
          DATA
                           ICRT/ 0 /
          DO 20 I=1,N
209:
210:
          XI = X(I)
211:
          DO 10 J=1.I
212:
          XIJ = XI - X(J)
213:
          F = FUNIT(XIJ, A, B, E, POIS, ITYPE)
```

```
214:
          FLEX(I,J) = F
215:
          FLEX(J,I) = F
216:
          FLXSTO(I,J) = F
217: 10 FLXSTO(J,I) = F
218: 20 CONTINUE
219:C...Invert the flexibility matrix to get the stiffness matrix
220:
          IERROR=0
221:
          CALL INVERT(FLXSTO, N, NMAX, ISTOR, STOR, IFLAG, STIF)
222:C...Make certain stiffness matrix is symmetric
223:
          DO 40 I=1.N
          DO 40 J=1,I
224:
225:
          SIJ = 0.5D0 * (STIF(I,J) + STIF(J,I))
226:
          STIF(I,J) = SIJ
227: 40 STIF(J,I) = SIJ
228: C... Check whether return from INVERT was normal
          IF (IFLAG .EQ. 2) IERROR = 1
229:
230:
          RETURN
231:
          END
232:CSSSSSSSSS FUNIT
          REAL*8 FUNCTION FUNIT(X,A,B,E,POIS,ITYPE)
233:
234: C... This function returns the deflection at position X due
235:C...to a unit distributed load at the origin. Parameter
236:C...ITYPE equals 1, 2, or 3 depending on whether a plane
237: C...strain, a plane stress, or a three-dimensional solution
238: C...is generated. E and POIS denote Young's modulus and
239:C...Poisson's ratio, respectively. In instances of plane
240: C...strain or plane stress, the unit load is distributed
241:C...over an area of width A and unit depth. The displace-
242:C...ment is adjusted to equal zero at X=B in the plane
243: C...case. In the instance of three-dimensional loading,
244: C... the unit load is distributed over a rectangular area
245:C...of width A and depth B. The displacement at X=infinity
246:C...vanishes for the three-dimensional case.
247:
          IMPLICIT REAL*8 (A-H,O-Z)
248:
          DATA
                          PI/ 3.14159265358979D0 /
249:
          IF (ITYPE .EQ. 3) GO TO 10
250:C...Plane stress
          C = 2.D0 / (PI * E * A)
252:C...Plane strain
          IF (ITYPE .EQ. 1) C = C * (1.D0 - POIS*POIS)
253:
254:
          FUNIT = -C * (F2D(DABS(X), .5DO*A) - F2D(B, .5DO*A))
255:
          RETURN
256:C...Three dimensional case
257: 10 T2 = (A + 2.D0 * DABS(X)) / B
          T1 = (-A + 2.D0 * DABS(X)) / B
258:
259: 20 FUNIT = (F3D(T2)-F3D(T1)) * (1.D0-POIS*POIS) / (PI*E*A)
260:
          RETURN
261:
          END
262:CS$$$$$$$$ F2D
263:
          REAL*8 FUNCTION F2D(U,V)
264:
          IMPLICIT REAL*8 (A-H,O-Z)
265:
          UA = DABS(U)
266:
          F2D = (UA+V) * DLOG(UA+V)
267:
          IF (UA .NE. V) F2D = F2D - (UA-V) * DLOG(DABS(UA-V))
```

```
RETURN
268:
269:
          END
270:C$$$$$$$$$ F3D
          REAL*8 FUNCTION F3D(T)
271:
272:
          IMPLICIT REAL*8 (A-H,O-Z)
273:
          F3D = DLOG(T+DSQRT(1.DO+T*T))
274:
          IF (T .EQ. O.DO) RETURN
275:
          S = 1.DO / DABS(T)
276:
          F3D = F3D + T * DLOG(S+DSQRT(1.DO+S*S))
277:
          RETURN
278:
          END
279:CSSSSSSSSS FACTR
          SUBROUTINE FACTR (A,N,NMAX,IPIVOT,SCAL,IFLAG)
280:
281: C... Perform triangular factorization of matrix A
282: C...using scaled row pivoting.
283:C...IFLAG = 1 means normal return
284:C...
              = 2 means matrix is singular
285:
          IMPLICIT REAL*8 (A-H,O-Z)
286:
          DIMENSION
                           A(NMAX,N), IPIVOT(N), SCAL(N)
287:
          IFLAG = 1
288:C...Initialize IPIVOT and SCAL
289:
          DO 20 I=1.N
290:
          IPIVOT(I) = I
291:
          ROWMAX = 0.DO
292:
          DO 10 J=1.N
293:
       10 ROWMAX = DMAX1(ROWMAX, DABS(A(I,J)))
          IF (ROWMAX .EQ. O.DO) GO TO 50
294:
295:
       20 SCAL(I) = ROWMAX
296:C...Perform Gauss reduction
297:
          NM1 = N - 1
298:
          IF (NM1 .EQ. O) RETURN
299:
          DO 40 K=1,NM1
300:
          J = K
301:
          KP1 = K + 1
          IP = IPIVOT(K)
302:
303:
          COLMAX = DABS(A(IP,K)) / SCAL(IP)
304:
          DO 30 I=KP1,N
305:
          IP = IPIVOT(I)
306:
          AWIKOV = DABS(A(IP,K)) / SCAL(IP)
307:
          IF (AWIKOV .LE. COLMAX) GO TO 30
308:
          COLMAX = AWIKOV
309:
          J = I
310:
       30 CONTINUE
          IF (COLMAX .EQ. O.DO) GO TO 50
3.1:
312:C
313:
          IPK = IPIVOT(J)
314:
          IPIVOT(J) = IPIVOT(K)
315:
          IPIVOT(K) = IPK
          DO 40 "=KP1.N
316:
317:
          IP = IPIVOT(I)
318:
          A(IP,K) = A(IP,K) / A(IPK,K)
319:
          RATIO = -A(IP,K)
320:
          DO 40 J=KP1,N
       40 A(IP,J) = PATIO * A(IPK,J) + A(IP,J)
```

321:

```
322:
          IF (A(IP,N) .EQ. O.DO) GO TO 50
323:
          RETURN
324:
       50 \text{ IFLAG} = 2
325:
          RETURN
326:
          END
327:CSSSSSSSSS BAKSUB
328:
          SUBROUTINE BAKSUB (A,N,NMAX,B,IPIVOT,X)
329:C...Solve simultaneous equations AX=B where matrix A has
330:C...been subjected to factorization by subroutine FACTR
331:
          IMPLICIT REAL*8 (A-H,O-Z)
332:
          DIMENSION
                           A(NMAX,N), B(N), IPIVOT(N), X(N)
333:
          IF (N .GT. 1) GO TO 10
334:
          X(1) = B(1) / A(1,1)
          RETURN
335:
       10 IP = IPIVOT(1)
336:
337:
          X(1) = B(IP)
          DO 30 K=2, N
338:
339:
          IP = IPIVOT(K)
340:
          KM1 = K - 1
341:
          SUM = 0.D0
342:
          DO 20 J=1,KM1
343:
       20 SUM = A(IP,J) * X(J) + SUM
       30 X(K) = B(IP) - SUM
344:
345:
          X(N) = X(N) / A(IP,N)
346:
          K = N
          DO 50 NP1MK=2, N
347:
          KP1 = K
348:
349:
          K = K - 1
350:
          IP = IPIVOT(K)
          SUM = 0.D0
351:
352:
          DO 40 J=KP1,N
       40 SUM = A(IP,J) * X(J) + SUM
353:
354:
       50 X(K) = (X(K)-SUM) / A(IP,K)
355:
          RETURN
356:
          END
357:CSSSSSSSSS INVERT
          SUBROUTINE INVERT( A, N, NMAX, IPIVOT, SCAL, IFLAG, AINV )
359:C...IFLAG = 2 indicates singular matrix
          IMPLICIT REAL*8 (A-H,O-Z)
360:
361:
          DIMENSION A(NMAX,1), AINV(NMAX,1), IPIVOT(1), SCAL(1)
          CALL FACTR( A, N, NMAX, IPIVOT, SCAL, IFLAG )
362:
363:
          IF( IFLAG .EQ. 2 ) RETURN
364:
          DO 10 J = 1, N
365:
     10 \quad SCAL(J) = 0.0D0
366:
          DO 20 I = 1, N
367:
          SCAL(I) = 1.0D0
368:
          CALL BAKSUB( A, N, NMAX, SCAL, IPIVOT, AINV(1,I) )
369:
          SCAL(I) = 0.0D0
370:
      20 CONTINUE
371:
          RETURN
372:
          END
373:C$$$$$$$$$$ F
          REAL*8 FUNCTION F( X, A )
375:C...Integral from 0 to X of LOG(ABS(X-A))-LOG(ABS(X+A))
```

```
376:
          IMPLICIT REAL*8 (A-H,O-Z)
377:
          FF = 0.D0
378:
          XMA = DABS(X) - A
379:
          XPA = DABS(X) + A
380:
          IF (XMA .EQ. O.DO) GO TO 10
381:
          FF = XMA * DLOG(DABS(XMA))
382: 10 FF = FF - XPA * DLOG(XPA)
          F = FF
383:
384:
          RETURN
385:
          END
386:CSSSSSSSSS STEP
387:
          REAL*8 FUNCTION STEP( X, A)
388:C...Unit step function.
389:
          IMPLICIT REAL*8 (A-H,O-Z)
390:
          STEP = 0.D0
          IF (X .GE. A) STEP = 1.DO
391:
392:
          RETURN
393:
          END
394:CSSSSSSSSS RAMP
          REAL*8 FUNCTION RAMP( X , A )
396: C... Linearly varying ramp load.
          IMPLICIT REAL*8 (A-H,O-Z)
397:
          RAMP = 0.D0
398:
          IF (X .GE. A) RAMP = X - A
399:
400:
          RETURN
401:
          END
402:C$$$$$$$$$ R
          REAL*8 FUNCTION R( X, A )
403:
404:C...Derivative of F(X.A)
405:
          IMPLICIT REAL*8 (A-H,O-Z)
          DATA ICRT / 0 /
406:
407:
          XMA = DABS(X-A)
408:
          IF (XMA .EQ. O.DO) GO TO 10
          R = DLOG(XMA)
409:
          XPA = DABS(X+A)
410:
411:
          IF (XPA .EQ. 0.DO) GO TO 10
412:
          R = R - DLOG(XPA)
413:
          RETURN
414: 10 WRITE( ICRT, 20 )
415: 20 FORMAT( /, ' ARGUMENT ERROR IN FUNCTION RAMP', / )
          STOP '
416:
417:
          END
418:C$$$$$$$$$$$$$$$$$$$$$
419:
          REAL*8 FUNCTION S( X, A )
420:C...Second derivative of F(X,A)
421:
          IMPLICIT REAL*8 (A-H,O-Z)
422:
          DATA ICRT / 0 /
423:
          IF (DABS(X) .NE. DABS(A)) GO TO 20
424:
          WRITE( ICRT, 10 )
                      ' ARGUMENT ERROR IN FUNCTION S', / )
425:
     10 FORMAT( /,
426:
          STOP '
427: 20 S = 2.D0 * A / (X * X - A * A)
          RETURN
428:
429:
          END
```

```
430:CSSSSSSSS G
         REAL*8 FUNCTION G( X, A )
431:
432:C...Integral of H(X,A)
433:
          IMPLICIT REAL*8 (A-H,O-Z)
434:
          G = -A
435:
          IF (X \cdot GT \cdot -A) \cdot G = X
436:
          IF (X .GT. A)
                         G = A
437:
          RETURN
438:
          END
439:C$$$$$$$$$ H
440:
          REAL*8 FUNCTION H( X, A )
441:C...Function equals 1 between -A and +A and equals zero otherwise.
442:
          IMPLICIT REAL*8 (A-H.O-Z)
443:
          H = 0.D0
          IF (DABS(X) .LE. A) H = 1.DO
444:
445:
          RETURN
          END
446:
447:CSSSSSSSS STFHVM
          SUBROUTINE STFHVM(X,N,N3MAX,E,POIS,WIDTH,DFIX,ITYP,STOR,ISTOR,
448:
                            IERROR, NOSTIF, FLXSTO, FLEX, STIF)
449:
450: C... This subroutine generates flexibility and stiffness matrices for
451:C...a half plane subjected to horizontal loads, vertical loads, and
452:C...couples applied at a selected set of coordinates on the surface.
453:C...
454: C... This routine calls the following routines: F, STEP, RAMP, R. S.
455:C...G, H, FACTR, BAKSUB, INVERT
456:
          IMPLICIT REAL*8 (A-H,O-Z)
457:
          DIMENSION
                          X(N), FLEX(N3MAX, N3MAX), STOR(1), ISTOR(1),
458:
         æ
                          FLXSTO(N3MAX,N3MAX), STIF(N3MAX,N3MAX)
459:
          DATA
                          PI / 3.14159265358979D0 /, ICRT/ 0 /
460:
          AH = 0.5DO * WIDTH
          FD = F(DFIX,AH)
461:
462:
          IF (ITYP .EQ. 2) GO TO 10
463:C...Plane strain
          CO = (1.DO - POIS ** 2) / (PI * AH * E)
464:
465:
          EO = (1.D0 + POIS) * (1.D0 - 2.D0 * POIS) / (2.D0 * AH * E)
          GO TO 20
466:
467:C...Plane stress
468: 10 CO = 1.DO / ( E * PI * AH )
469:
          EO = (1.DO - POIS) / (2.DO * E * AH)
470:
     20
         CONTINUE
471:
          DO 40 I = 1. N
          DO 30 J = 1, N
472:
473:
          XIJ = X(I) - X(J)
474:
          FIJ = CO * (F(XIJ,AH) - FD)
          GIJ = EO * G(XIJ, AH)
475:
          HIJ = EO * H(XIJ, AH)
476:
477:
          RIJ = CO * R(XIJ, AH)
          SIJ = EO * S(XIJ, AH)
478:
479:
          IR = 3 * I - 2
          JC = 3 * J - 2
480:
481:
          FLEX(IR,JC) = FIJ
          FLEX(IR+1,JC) = GIJ
482:
483:
          FLEX(IR+2,JC) = HIJ
```

```
484:
          FLEX(IR,JC+1) = -GIJ
485:
          FLEX(IR+1,JC+1) = FIJ
486:
          FLEX(IR+2,JC+1) = RIJ
487:
          FLEX(IR,JC+2) = HIJ
488:
          FLEX(IR+1,JC+2) = -RIJ
489:
          FLEX(IR+2,JC+2) = -SIJ
490: 30 CONTINUE
491: 40 CONTINUE
492:
          N3 = 3 * N
          DO 50 I=1,N3
493:
494:
          DO 50 J=1,I
         FSTO = FLEX(I,J)
495:
496:
          FLXSTO(I,J) = FSTO
497: 50 FLXSTO(J,I) = FSTO
498:C...Invert the flexibility matrix if stiffness is required
499:
          IF (NOSTIF .EQ. 1) RETURN
500:
          IERROR = 0
          CALL INVERT(FLXSTO, N3, N3MAX, ISTOR, STOR, IFLAG, STIF)
501:
502:C...Make sure stiffness matrix is symmetric
503:
          DO 60 I=1,N3
          DO 60 J=1,I
504:
505:
          SIJ = 0.5D0 * (STIF(I,J) + STIF(J,I))
506:
          STIF(I,J) = SIJ
507: 60 STIF(J,I) = SIJ
508:C...Check whether return form INVERT was normal
509:
          IF (IFLAG .EQ. 2) IERROR = 1
510:
          RETURN
511:
          END
```